FORMAL LANGUAGE & AUTOMATA THEORY

NOTE:

WBUT course structure and syllabus of 4th semester has been changed from 2020. The syllabus of **FORMAL LANGUAGE & AUTOMATA THEORY h**as been completely redesigned and restructured in present curriculum. Taking special care of this matter we are providing the relevant MAKAUT university solutions and some model questions & answers for newly introduced topics, so that students can get an idea about university questions patterns.

INTRODUCTION

☞ Chapter at a Glance

Alphabets: An alphabet Σ is a finite set of symbols.
Example:

$$\Sigma_1 = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,$$
$$o,p,q,r,s,t,u,v,w,x,y,z\}$$

Recursive Languages, recursive sets: The recursive languages and recursive sets are those languages and sets accepted by Turing machines and unrestricted grammars.
Recursively enumerable set: The sets, languages, that can be generated (enumerated) where all strings in the set, language, of a given length can be generated. If both a set and its complement are recursively enumerable, then the set is recursive.

Chomsky hierarchy of grammars /languages

***Type 0*, Grammar** is any phrase structure grammar without any restrictions.
Example: In abAbcd->abABbcd, ab is the left context, bcd is the right context, α=ab.
Type 1, Grammars is a production of the form ϕAψ->ϕα ψ is called type 1 production if α ≠ ^.
In type 1 production erasing of A is not permitted.
A grammar is called Type 1 or context sensitive or context dependent if all the productions are Type 1 production. The production S->^ is also allowed in a Type 1 grammar, but in this case S does not appear on the R.H.S of any production.
Example: In aAbcD->abcDbcD is the type 1 production. a, bcD are the left context, and right context.

Type 2, Grammars is a production of the form A->α ,where A,B∈V_N and α ∈$(V_N \cup \Sigma)^*$. In other words, the L.H.S has no left context or right context.
A grammar is called Type 2 or context free grammar if all the productions are Type 2 productions.
Example: (c) S ->Aa, A ->a, B ->abc, A ->^, are type 2 productions.

Type 3, Grammars is a production S -> ^ is allowed in type 3 grammar, but in this case S does not appear on the right-hand side of any production.
A grammar is called Type 3 or regular grammar if all the productions are Type 3 productions.
Example: (d) S ->aS, A ->a, A ->aB, are type 3 productions.

Language: A language L over an alphabet A is a collection of words on A, A^* denotes the set of all words on A. Thus a language L is simply a subset of A^*.

Grammar: A grammar G is defined as a quadratuple G = (V, T, S, P),
Where V is a finite set of object called variables, T is a finite set of objects called terminal symbols.

S ∈ V is a special symbol called the start

P is a finite set of productions.

e.g., G = ({S}, {a, b}, S, P}
with P given by

$S \rightarrow aSb$

$S \rightarrow \lambda$

Then $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

So we can write $S \Rightarrow aabb$.

The string aabb is a sentence in the language generated by G, while aaSbb is a sentential form.

Multiple Choice Type Questions

1. The basic limitation of Finite State Machine of that [WBUT 2007, 2010]
a) it cannot remember arbitrary large amount of information
b) it cannot recognize grammars that are regular
c) it sometimes recognize grammars that are not regular
d) all of these
Answer: (a)

2. A shift register is a [WBUT 2010]
a) Mealy M/c b) Moore M/c c) Turing M/c d) All of these
Answer: (d)

3. FSM can recognize [WBUT 2011]
a) a grammar dependent on characteristic of FSM b) on CFG
c) any unambiguous grammar d) only regular grammar
Answer: (d)

4. Which is true of the following? [WBUT 2013, 2019]
a) Merger graph is directed graph b) Compatible graph is directed graph
c) Both are directed d) None of these
Answer: (b)

5. Compatible pairs are obtained from [WBUT 2014, 2015, 2017]
a) Merger Graph b) Compatible Graph
c) Testing Table d) Testing Graph
Answer: (a)

6. A finite automata recognizes [WBUT 2018]
a) Any Language b) Context Sensitive Language
c) Context Free Language d) Regular Language
Answer: (d)

7. Maximum number of states of a DFA converted from a NFA with n states is **[WBUT 2018]**

a) n **b)** n^2 **c)** 2^n **d) None of these**

Answer: (c)

8. Regular Expression is accepted by **[WBUT 2018]**

a) Finite Automata **b) Push Down Automata**

c) Turing Machine **d) All of these**

Answer: (d)

9. The production grammar $\{S \to aSbb,\ S \to abb\}$ is **[WBUT 2019]**

a) Type-3 grammar **b) Type-2 grammar**

c) Type-1 grammar **d) Type-0 grammar**

Answer: (b)

10. The reduced grammar of $S \to AB|a, A \to a$ is **[WBUT 2019]**

a) $S \to a,\ A \to a$ **b)** $S \to a|A,\ A \to a$

c) $S \to a$ **d)** $S \to aa$

Answer: (c)

11. What is the highest type number to the grammar given by the following production rules **[MODEL QUESTION]**

S→Aa, A→c|Ba, B→abc

a) zero **b) one** **c) two** **d) three**

Answer: (d)

12. If $G = (\{S\}, \{a\}, \{S \to SS,\} S)$, the languages generated by G is **[MODEL QUESTION]**

a) L (G) = ϕ **b) L (G) = a^n**

c) L (G) = a ϕ **d) L (G) = a^n b a^n**

Answer: (a)

13. Which of the following grammars generates strings with any number of 1's? **[MODEL QUESTION]**

a) $S \to 1A,\ A \to \varepsilon$ **b)** $S \to 1S,\ S \to \varepsilon$

c) $S \to S1,\ S \to \varepsilon$ **d) (b) & (c)**

Answer: (d)

14. Given a grammar G a production of G with a dot at some position of the right side is called **MODEL QUESTION]**

a) LR (1) item of G **b) LR (0) item of G**

c) Both (a) and (b) above **d) None of these**

Answer: (b)

15. Which of the choice in an operator grammar equivalent for $S \rightarrow SAS/a$, $A \rightarrow bsb \mid b$ Assume S is the start symbol **[MODEL QUESTION]**

a) $S \rightarrow SbAbS \mid a, A \rightarrow b$
b) $S \rightarrow SbSbS \mid SbS \mid a$
c) $S \rightarrow SAS \mid a, A \rightarrow bsb \mid b$
d) $S \rightarrow SbS \mid b$

Answer: (b)

16. Grammar $S \rightarrow aAb.A \rightarrow aAb \mid a$ is in **[MODEL QUESTION]**

a) LR (1) not in LR (0)
b) LR (0) but not in LR (1)
c) Both LR (0) and LR(1)
d) Neither in LR (0) and LR (1)

Answer: (a)

17. What is the language of the grammar with the following production rules? **[MODEL QUESTION]**

$$S \rightarrow ASb \mid c$$
$$A \rightarrow a$$

a) $\{a^n cb^n \mid n \in \mathrm{IN}\}$
b) $\{xcb \mid x \in \{a\}^*\}$
c) $\{acy \mid y \in \{b\}^*\}$
d) None of these

Answer: (a)

18. A grammar has the following production:

$$S \rightarrow aSSb \mid a \mid bSa$$

Which of the following sentences are in the language that is generated by this grammar? **[MODEL QUESTION]**

a) aaaaabb
b) aabbaabb
c) bbbaabbaa
d) All of these

Answer: (a)

19. Take a look at the following grammar:

$$S \rightarrow AaC \mid Bd$$
$$A \rightarrow BC$$
$$B \rightarrow bB \mid C$$
$$C \rightarrow accS \mid$$

For which non-terminals N is symbol a part of the collection follow N? Give the best answer. **[MODEL QUESTION]**

a) $\{A\}$
b) $\{A, C\}$
c) $\{A, B, C\}$
d) $\{A, B, C, S\}$

Answer: (a)

Short Answer Type Questions

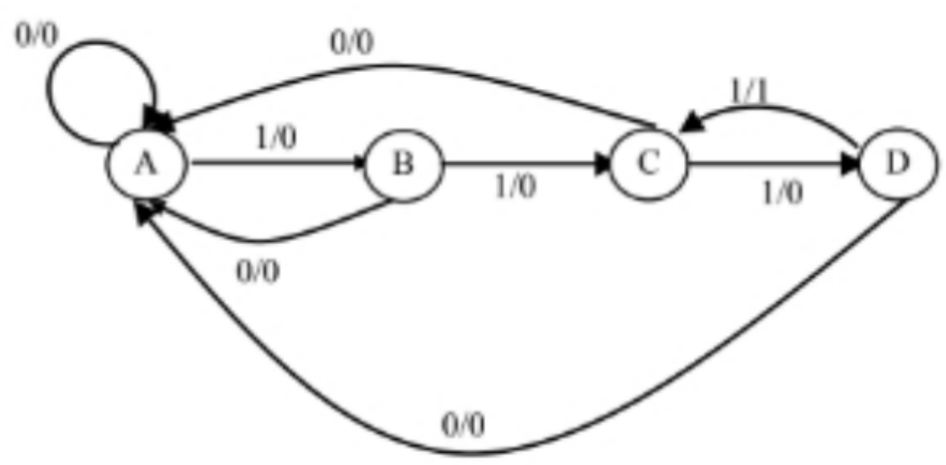
1. A long sequence of pulses enters a two-input, two-output synchronous sequential circuit, which is required to produce an output pulse Z = 1 whenever the sequence 1111 occurs. Overlapping sequences are accepted; for example, if the input is 01011111……., the required output is 00000011……..

i) Draw a state diagram.

ii) Select the assignment and show the excitation and output tables.

[WBUT 2004, 2008]

Answer:



Present State	*Next State, Z*	
	Input x = 0	*Input x = 1*
A	*A,0*	*B,0*
B	*A,0*	*C,0*
C	*A,0*	*D,0*
D	*A,0*	*C,1*

	Y_1Y_2		Z	
y_1y_2	x=0	x=1	x=0	x=1
A→00	00	01	0	0
B→01	00	11	0	0
C→11	00	10	0	0
D→10	00	11	0	1

2. What are the limitations of sequential circuit? **[WBUT 2009]**

Answer:

A sequential circuit can only represent a finite-state machine. In such a machine, an arbitrarily long sequence of input symbols is bound to create a periodic output with a period not more than the number of states in the machine. Thus, computations which needs to remember an arbitrary number of past input symbols, for example in recognizing $a^n b^n, n > 0$, cannot be implemented by sequential machines. By the same reasoning, a sequential machine cannot multiply two arbitrary length binary strings.

3. Define: (i) Alphabet **[MODEL QUESTION]**
(ii) Recursive languages
(iii) Languages

(iv) Regular languages

Answer:

(i) Alphabets:

An alphabet Σ is a finite set of symbols.

Example:

$$\Sigma_1 = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,$$
$$o,p,q,r,s,t,u,v,w,x,y,z\}$$

Example:

$$\Sigma_2 = \{0,1\}$$

Example:

The set of lexical elements of a programming language (keywords, syntax, identifiers etc.).

(ii) Recursive Languages:

The recursive languages and recursive sets are those languages and sets accepted by Turing machines and unrestricted grammars.

Recursively enumerable sets, r.e. languages

The sets, languages, that can be generated (enumerated) where all strings in the set, language, of a given length can be generated. If both a set and its complement are recursively enumerable, then the set is recursive.

(iii) Languages:

A language L over an alphabet A is a collection of words on A, A^* denotes the set of all words on A. Thus a language L is simply a subset of A^*.

e.g., $\Sigma = \{a, b\}$ then

$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aab, bbb \ldots\}$

The set

$\{a, aa, aab\}$

is a language on Σ. Because it has a finite number of sentences, we call it a finite language. The set

$L = \{a^n b^n : n \geq 0\}$

is also a language on Σ. The string aabb and aaabbb are in the language L, but the string abb is not in L. This language is infinite.

(iv) Regular Languages:

Let L be a language over A. then L is called a regular language over A. if there exists a regular expression r over A such that L = L(r).

4. a) Write down the operations of languages. **[MODEL QUESTION]**

b) Write down the applications of Grammars.

Answer:

a) The operations of Languages are:

(i) Union: $L_1 \cup L_2$

(ii) Intersection: $L_1 \cap L_2$

(iii) Complementation: $\overline{L} = \Sigma^* - L$

(iv) Concatenation: $L_1 L_2 = \{uv \mid u \in L_1, v \in L_2\}$

b) The applications of Grammar are:

- Specifying syntax of programming language
- Representing syntactic structures in natural languages
- Models of computation

Context-free grammars are the most commonly used kind of grammar in computer science.

5. What do you mean by Distinguishable and Indistinguishable states? **[MODEL QUESTION]**

Answer:

Two distinct states, Si, and S_j , of a machine *M* are *distinguishable* if and only if there exists at least one finite input sequence which, when applied to *M*, causes different output sequences, depending on whether S_i or S_j is the initial state.

The sequence which distinguishes these states is called a distinguishing sequence of the pair (S_i, S_j).

If no distinguishing sequence exists for the pair (S_i, S_j), then the states are said to be indistinguishable.

The principle of state minimization of a finite state machine is based on the principle of identifying equivalent classes for states which are indistinguishable in the original machine and then assigning one state per class in the reduced machine.

6. If G = ({S}, {0, 1}, {S→0S1, S→ ^}, S, find L(G). **[MODEL QUESTION]**

Answer:

As S→ ^ is a production, S⇒ ^. So ^ is in L(G). Also, for all n ≥ 1.

$$S \underset{G}{\Rightarrow} 0S1 \underset{G}{\Rightarrow} 0^2S1^2 \underset{G}{\Rightarrow} \cdots \Rightarrow 0^nS1^n \Rightarrow 0^n1^n$$

Therefore, $0^n1^n \in L(G)$ for $n \geq 0$

Hence, $\{0^n1^n \mid n \geq 0\} \subseteq L(G)$

To show that $L(G) \subseteq \{0^n1^n \mid n \geq 0\}$, we start with w in L(G). The derivation of w starts with S. If S→ ^ is applied first, we get ^. In this case w = ^. Otherwise the first production to be applied is S→0S1. At any stage if we apply S→ ^, we get a terminal

string. Also, the terminal string is obtained only by applying $S \rightarrow \wedge$. Thus the derivation of w is of the form $S \underset{G}{\overset{*}{\Rightarrow}} 0^n S 1^n \underset{G}{\Rightarrow} 0^n 1^n$, i.e., $L(G) \subseteq \{0^n 1^n \mid n \geq 0\}$.

Therefore, $L(G) = \{0^n 1^n \mid n \geq 0\}$

7. For a grammar G the set of productions P is $S \rightarrow aA$, $A \rightarrow bbA$, $A \rightarrow c$. Describe the language L(G). [MODEL QUESTION]

Answer:

$$S \Rightarrow aA \Rightarrow abbA \Rightarrow abbbbA = ab^4 A \Rightarrow ....$$
$$\Rightarrow ab^{2n} A \Rightarrow ab^{2n} c$$
$$\therefore L(G) = \{ab^{2n} c / n \geq 0\}.$$

Long Answer Type Questions

1. Draw the Merger graph, Merger Table, Compatibility graph and then minimize the following: [WBUT 2011, 2016]

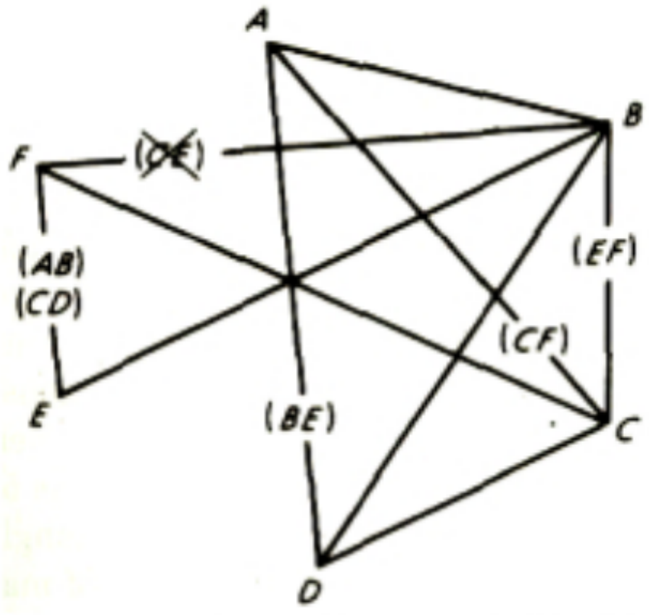
Present State	Next State, o / p			
	i/p = 0	i/p = 1	i/p = 2	i/p = 3
A	–	C, 1	E, 1	B, 1
B	E, 0	–	–	–
C	F, 0	F, 1	–	–
D	–	–	B, 1	–
E	–	F, 0	A, 0	D, 1
F	C, 0	–	B, 0	C, 1

Answer:

Step 1: Draw the arc of compatible pair. Compatible pair means that two present state entries of the next state should not conflict.

Here AB, BC, EF, etc are non-conflicting pair.

On other hands DE is a conflicting pair, because output of I_3 is conflicting i.e. B consist the next state output of I_3 is 1 whereas E contain 0. Similarly CE, DF are also conflicting so these are non-compatible pair.



Note: now the merger graph displays all possible pair of state and their implied pair, and since a pair of state is compatible only if its implied pair is.

Step 2: Check and determine whether the implied pair are indeed compatibles. A pair $(S_p S_q)$ *is* incompatible if no arc is drawn between vertices S_p and S_q. In such a case, if $(S_p S_q)$ *is* written space of an interrupted arc, the entry $(S_p S_q)$ is crossed off, and the, corresponding arc is ignored.

Here the condition for *(BF)* to be compatible is that *(CE)* be compatible, but since there is no arc drawn between C and *E, (CE)* is incompatible and the arc between *B* and *F* is ignored. Thus states *B* and *F* are incompatible. Next, it is necessary to check whether the incompatibility of *(BF)* does not invalidate any other implied pair, that is, if *(BF)* is not written in the space of another interrupted arc, and so on. The interrupted arcs which remain in graph, after all the implied pairs have been verified to be compatible are regarded as solid ones.

The merger graph of Machine *M* reveals the existence of compatible pairs:

(AB),(AC)(AD),(BC),(BD),(BE),(CD),(CF),(EF)

In order to find a minimal set of compatibles, which covers original machine and can be used as a basis for the construction of the minimal machine, it is often useful to find the set of maximal compatibles Recall that a compatible is maximal if it is not contained in any other compatible. In terms of the merger graph, we are looking for complete polygons which are not contained within any higher-order polygons. [A *complete polygon* is one in which all possible (n-3) n/2 diagonals exist, where *n* is the number of sides in the polygon.] Since the states covered by a complete polygon are all pair wise compatible they constitute a compatible; and if the polygon is not contained; higher-order complete polygon, they constitute a maximal compatible.

In the merger graph the set of highest-order polygons are the *(ABCD)* and the arcs *(CF), (BE),* and *(EF).* Generally, after a complete polygon of order *n* has been found, all polygons of order n –1 contained in it can be ignored. Consequently, the triangles *(ABC),* etc., are not considered. Thus the following set of maximal compatibles for machine *M* is:

{(ABCD),(BE),(CF),(EF)}

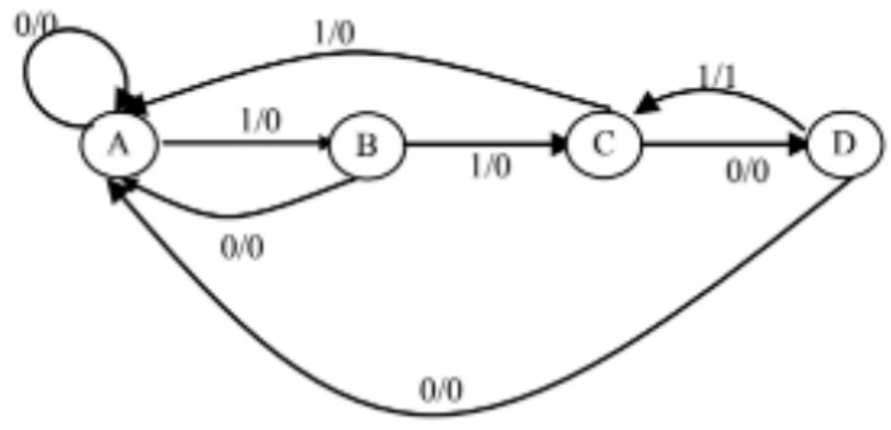
2. A long sequence of pulses enters a synchronous sequential circuit, which is required to produce an output pulse $z=1$ whenever the sequence 1001 occurs. Overlapping sequences are accepted.

i) Draw a state diagram

ii) Select an assignment and show the excitation and output tables. [WBUT 2015]

Answer:

a) i)

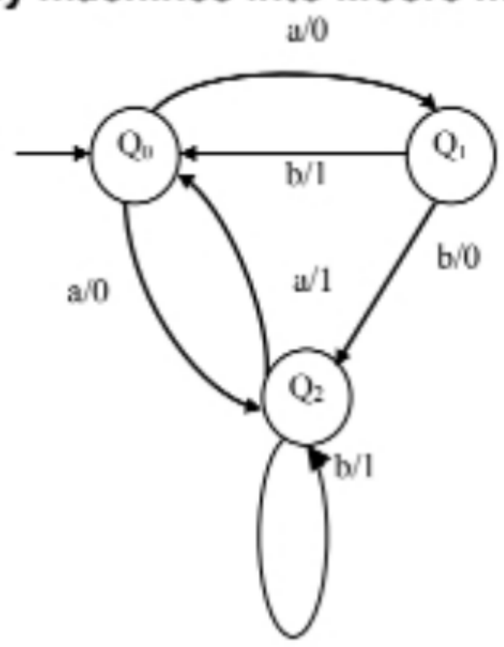


ii)

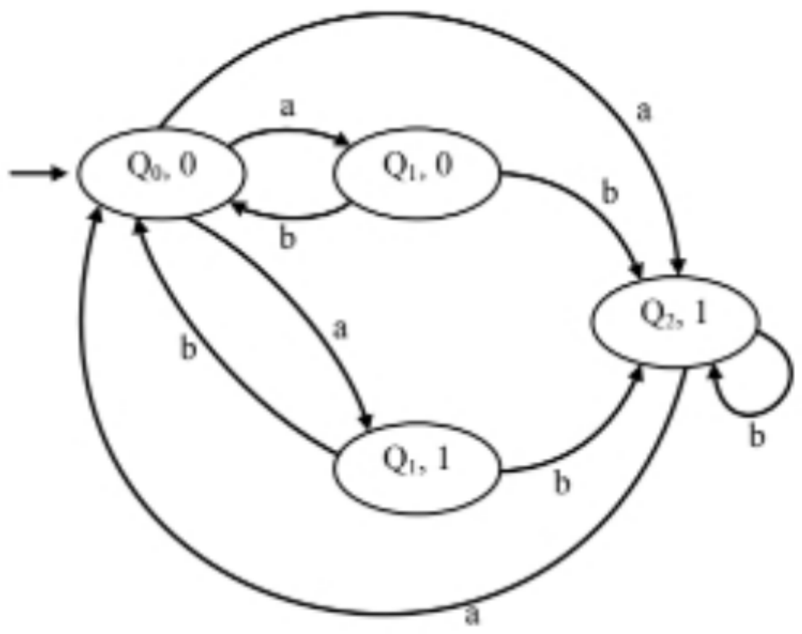
Present State	*Next State, Z*	
	Input x = 0	*Input x = 1*
A	*A,0*	*B,0*
B	*A,0*	*C,0*
C	*D,0*	*A,0*
D	*A,0*	*C,1*

	Y_1Y_2		Z	
y_1y_2	x=0	x=1	x=0	x=1
A→00	00	01	0	0
B→01	00	11	0	0
C→11	10	00	0	0
D→10	00	11	0	1

3. Convert the following Mealy machines into Moore machine. **[WBUT 2015]**



Answer:



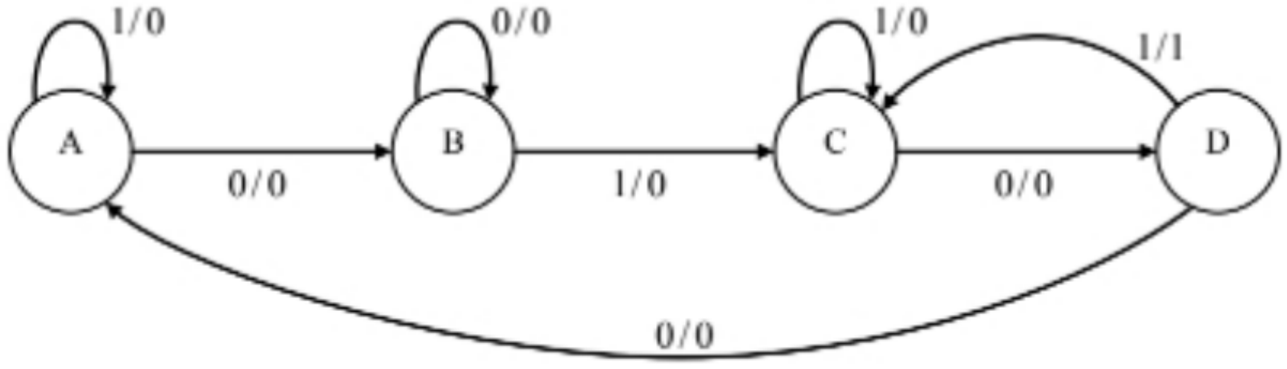
4. Draw sequence detector diagram, table, k-map and circuit for the sequence 0101 which generates a 1 as output after every 0101 sequence. Consider overlapping of sequences. **[WBUT 2018]**

Answer:

Sequence detector:

i/p: 0 1 0 1 0 1 0 1 .....

o/p: 0 0 0 1 0 1 0 1 .....



Diagram

Present State	Next State	
	I/P = 0	I/P = 1
A	B, 0	A, 0
B	B, 0	C, 0
C	D, 0	C, 0
D	A, 0	C, 1

Table

∴ **K-map:**

Let A → 00, B → 01, C → 10, D → 11

Present State		Next State			
		i/p = 0		i/p = 1	
	$y_1\ y_2$	$y_1\ y_2$	z_1	$y_1\ y_2$	z_2
A	0 0	0 1,	0	0 0,	0
B	0 1	0 1,	0	1 0,	0
C	1 0	1 1,	0	1 0,	0
D	1 1	0 1,	0	1 0,	1

$\therefore$

For i/p = 0

y_2 \ y_1	0	1
0	0	0
1	0	0

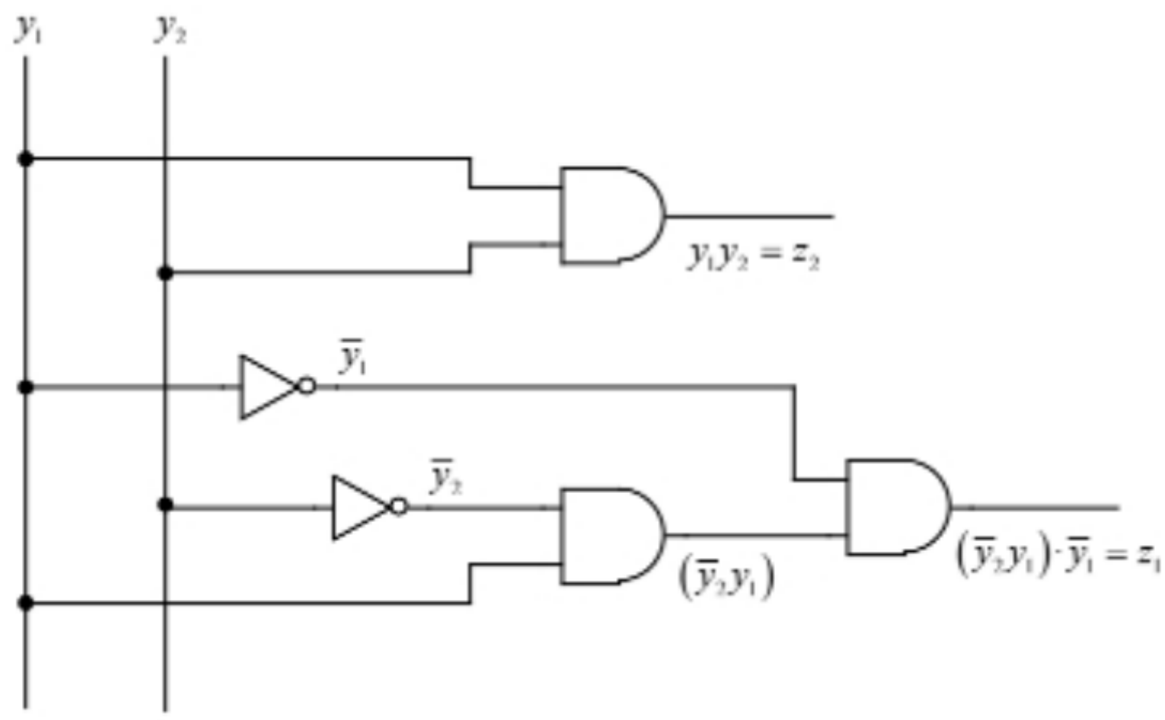
$z_1 = (\bar{y}_2\, y_1)\cdot\bar{y}_1$

For i/p = 1

y_2 \ y_1	0	1
0	0	0
1	0	1

$z_2 = y_1\, y_2$

Circuit:



5. Draw the merger graph, merger table, compatibility graph and then find the minimal closed covering with justification of the following machine:

PS	NS, o/p		NS, o/p	
	i/p = 0	i/p = 1	i/p = 2	i/p = 3
A	–	C, 1	E, 1	B, 1
B	E, 0	–	–	–
C	F, 0	F, 1	–	–, 1
D	–	–	B, 1	–
E	–	F, 0	A, 0	D, –
F	C, –	–	B, 0	C, 1

Answer:

Similar to Long Answer Type Questions No. 1 of Chapter 1.

6. Define: Chomsky hierarchy of Languages. **[MODEL QUESTION]**

Answer:

Type 0, Grammar is any phrase structure grammar without any restrictions.

To define the other types we need a definition.

In a production of the form $\phi A\psi \rightarrow \phi\alpha\ \psi$, where A is a variable, ϕ is called the left context ψ, the right context and $\phi\alpha\ \psi$ the replacement string.

Example: (a) i) In abAbcd->abABbcd, ab is the left context, bcd is the right context, α=ab

ii) In AC->A, A is the left context, ^ is the right context. α=^ The production simply erases C when the left context is A and the right context is ^.

iii) a) In C-> ^, the left and right context are ^.α=^ The production simply erases C when in any context.

Type 1, Grammars is a production of the form $\phi A\psi \rightarrow \phi\alpha$ ψ is called type 1 production if $\alpha \neq$ ^. In type 1 production erasing of A is not permitted.

A grammar is called ***Type 1*** or ***context sensitive*** or ***context dependent*** if all the productions are Type 1 production. The production S->^ is also allowed in a Type 1 grammar, but in this case S does not appear on the R.H.S of any production.

Example: (b) i) In aAbcD->abcDbcD is the type 1 production. a, bcD are the left context, and right context.

ii) AB -> AbBc is a type 1 production. The left context is A and right context is ^.

iii) A-> abA is a type 1 production. The both left and right context is ^.

Type 2, Grammars is a production of the form A->α ,where A,B$\in V_N$ and $\alpha \in (V_N \cup \Sigma)^*$. In other words, the L.H.S has no left context or right context.

A grammar is called ***Type 2*** or ***context free grammar*** if all the productions are Type 2 productions.

Example: (c) S ->Aa, A ->a, B ->abc, A ->^, are type 2 productions.

Type 3, Grammars is a production S -> ^ is allowed in type 3 grammar, but in this case S does not appear on the right-hand side of any production.

A grammar is called ***Type 3*** or ***regular grammar*** if all the productions are Type 3 productions.

Example: (d) S ->aS, A ->a, A ->aB, are type 3 productions.

Language: A language L over an alphabet A is a collection of words on A, A^* denotes the set of all words on A. Thus a language L is simply a subset of A^*.

e.g., $\Sigma = \{a, b\}$ then

$\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aab, bbb \ldots\}$

The set

{a, aa, aab}

is a language on Σ. Because it has a finite number of sentences, we call it a finite language. The set

$L = \{a^n b^n : n \geq 0\}$

is also a language on Σ. The string aabb and aaabbb are in the language L, but the string abb is not in L. This language is infinite.

REGULAR LANGUAGES & FINITE AUTOMATA

☞ Chapter at a Glance

Deterministic Finite State Automata: (DFA) An automaton, a language recognition device, can also be defined rigorously. A deterministic finite state automaton is a quintuple $M = <q,i,\delta ,q_0,f>$; Where:

- Q is a finite set of states;
- I is a finite set of input symbols;
- δ is the next-state function $\delta : Q \times I \rightarrow Q$;
- $q_0 \in Q$ is the initial state;
- $F \subseteq Q$ is the set of final states (or accepting states).

The function $\delta: Q \times I \rightarrow Q$ can be extended to the domain $Q \times I^*$, defining the function δ^* as follows: $\delta^*(q,\lambda)=q$ for each $q \in Q$

$\delta^*(q,xa)=\delta(\delta^*(q,x),a)$ for each $x \in I^*$, $a \in I$. The language L(A) accepted or recognized by the automaton A is defined as the set of words accepted by A.

Non-deterministic Finite Automata (NFA): A nondeterministic finite automata is a quintuple $M= <Q,I,\delta,q_0,F>$; *where:*

- Q is a finite set of states;
- I is a finite set of input symbols;
- δ is the, possibly partial, transition function
- $\delta: Q \times (I \cup \{\lambda\}) \rightarrow 2^Q$
- $q_0 \in Q$ is called the initial state;
- $F \in Q$ is called the set of final states

Myhill-Nerode theorem and minimization to eliminate useless states.

The **Myhill-Nerode Theorem** says the following three statements are equivalent:

The set L, a subset of Sigma star, is accepted by a DFA. (We know this means L is a regular language.)

L is the union of some of the equivalence classes of a right invariant (with respect to concatenation) equivalence relation of finite index.

Let equivalence relation RL be defined by: x RL y if and only if for all $z \in \Sigma^*$ star, $xz \in L$ exactly when $yz \in L$. Then RL is of finite index.

Regular Expressions/Grammars:

A ***regular expression E*** over I denotes a language L(I) over I. The syntax of regular expressions is defined inductively together with the languages they denote:

1. The constants **ε** and **Φ** are regular expressions, with L(**ε**) = {ε} and L(**Φ**) = Φ.
2. For each a in I, **a** is a regular expression, with L(**a**) = {a}.
3. If E and F are regular expressions, then (**E** + **F**) is a regular expression, with L(**E** + **F**) = L(**E**)∪L(**F**).
4. If E and F are regular expressions, then (**EF**) is a regular expression, with L((**EF**)) = L(**E**)L(**F**).
5. If *E* is a regular expression, then (**E***) is a regular expression, with L((**E***)) = (L(**E**))*.

Precedence rules for saving parentheses: * has highest precedence, then concatenation, then +.

Pumping Lemma for Regular Language: The Pumping Lemma is generally used to prove a language is not regular.

Multiple Choice Type Questions

1. Choose the correct statements: **[WBUT 2006, 2012]**

a) moore and mealy machine are FSM with output capability
b) any given Moore machine has an equivalent Mealy machine
c) any given Mealy machine has an equivalent Moore machine
d) moore machine is not an FSM

Answer: (a), (b) and (c)

2. Pumping Lemma is generally used for proving whether **[WBUT 2006, 2016]**

a) a given grammar is regular
b) a given grammar is not regular
c) two given grammars are equivalent or not
d) none of these

Answer: (b)

3. A solution to the equation $R = Q + RP$ is **[WBUT 2007, 2013, 2017, 2019]**

a) $R = QP^*$ **b)** $Q = RP^*$ **c)** $P = RQ^*$ **d)** $R = PQ^*$

Answer: (a)

4. Which of the following sets is regular? **[WBUT 2007, 2012]**

a) $\{a^i : i = n^2, n >= 1\}$ **b)** $\{a^p : p \text{ is a prime}\}$

c) $\{ww : w \text{ is in } \{a, b\}^*\}$ **d)** $\{a^{2n} : n >= 1\}$

Answer: (d)

5. The value of $L(\phi^*)$ is **[WBUT 2007, 2019]**

a) $\sum$ **b)** $\{\varepsilon\}$ **c)** $\{\ \}$ **d) none of these**

Answer: (c)

6. The regular expression representing the set of all strings over $\{x, y\}$ ending with *xx* beginning with *y* is **[WBUT 2007, 2012]**

a) $xx(x+y)^*y$ **b)** $yy(x+y)^*x$ **c)** $y(x+y)^*xx$ **d)** $y(xy)^*xx$

Answer: (d)

7. Regular expression $(a\,|\,b)(a\,|\,b)$ denotes the set **[WBUT 2007, 2012]**

a) $\{a, b, ab, aa\}$ **b)** $\{a, b, ba, bb\}$ **c)** $\{a, b\}$ **d)** $\{aa, ab, ba, bb\}$

Answer: (d)

8. Palindromes cannot be recognized by any FSM because **[WBUT 2007, 2012]**

a) an FSM cannot remember arbitrary, large amount of information
b) an FSM cannot deterministically fix the mid-point
c) even or the mid-point is known, an FSM cannot find whether the second half of the string matches the first half
d) all of these

Answer: (c)

9. If S is the number of states in NDFA then equivalent DFA can have maximum of **[WBUT 2008, 2019]**

a) S states **b) S-1 state** **c) 2^Sstates** **d) 2^S-1 states**

Answer: (c)

10. DFA has **[WBUT 2008]**

a) single final state
b) more than one initial states
c) unique path (for a set of inputs) to the final state
d) all of these

Answer: (c)

11. What is the highest type number to the grammar given by the following production rules S→Aa, A→c|Ba, B→abc **[WBUT 2008]**

a) zero **b) one** **c) two** **d) three**

Answer: (d)

12. Given an arbitrary NDFA with *n* states, the maximum number of states in an equivalent minimized DFA is at least **[WBUT 2008]**

a) n^0 **b) 2^n** **c) n!** **d) None of these**

Answer: (d)

13. Which of the following regular expressions over {0, 1} denotes the set of all strings not containing 100 as a sub-string? **[WBUT 2008]**

a) 0*(1*0)* **b) 0*1010*** **c) 0*1*01*** **d) 0*(10+1)***

Answer: (d)

14. Which of the following is regular? **[WBUT 2008, 2019]**

a) Strings of 0's, whose length is a perfect square
b) Strings of all palindromes made up of 0's & 1's
c) Strings of 0's, whose length is a prime number
d) Strings of odd number of zeroes

Answer: (d)

15. $a^*(a+b)^*$ **is equivalent to** **[WBUT 2009, 2013]**

a) a^*+b^* b) $(ab)^*$ c) a^*b^* d) None of these

Answer: (d)

16. Which is true of the following? **[WBUT 2009]**

a) Merger graph is a directed graph
b) Compatible graph is a directed graph
c) both are directed
d) None of these

Answer: (b)

17. Merger table is a substitute of **[WBUT 2009]**

a) Merger graph
b) Compatible graph
c) Minimized machine
d) Finite state machine

Answer: (a)

18. The loop-free testing graph indicates that **[WBUT 2010]**

a) the machine has finite memory
b) the machine has non-finite memory
c) the machine has finite states
d) the machine has non-finite states

Answer: (a)

19. Input sequence of an information lossless machine can be determined from the knowledge of **[WBUT 2010]**

a) only output sequence
b) output sequence and initial state
c) output sequence, initial state and final state
d) initial state

Answer: (a)

20. Consider the following regular expression: $R=(ab+abb)^*bbab$

Which of the following is not in the set denoted by R? **[WBUT 2010]**

a) ababab b) ababbabbbab c) abbbab d) abbabbbab

Answer: (a)

21. Which of the following is correct? **[WBUT 2010, 2016]**

a) Language can be derived from the FA
b) Regular expressions can be derived from the FA
c) FA can be derived from the language
d) Both (a) & (b)

Answer: (d)

22. DFA converted from an NFA with *n* states can have maximum **[WBUT 2009]**

a) n **states** **b)** $n!$ **states** **c)** 2^n **states** **d)** ${}^nC_2$ **states**

Answer: (c)

23. The following production rules of a regular grammar generates a language L

$$S \to aS / bS / a / b$$

The regular expression for L is **[WBUT 2011]**

a) $a+b$ **b)** $(a+b)$ **c)** $(a+b)(a+b)^*$ **d)** $(aa+bb)a^*b^*$

Answer: (b)

24. Moore machine output depends on **[WBUT 2011]**

a) input
b) input and present state
c) present state
d) none of these

Answer: (c)

25. DFA has a transition function **[WBUT 2011]**

a) $Q \times \Sigma$ **to** Q **b)** $Q \times \Sigma$ **to** 2^Q **c) both (a) and (b)** **d) none of these**

Answer: (a)

26. If Q **is the number of states in the NFA, the equivalent DFA can have maximum number of states** **[WBUT 2011]**

a) Q **b)** $Q-1$ **c)** $2Q-1$ **d)** 2^Q

Answer: (d)

27. What is the RE for the language set strings with at least one 1, one 2 and one 3? **[WBUT 2012]**

a) 1+2+3 **b) 11* 22* 33*** **c) 1* 2* 3*** **d) both (a) and (b)**

Answer: (b)

28. The basic limitation of FSM is that **[WBUT 2012, 2016]**

a) it can't remember arbitrary large amount of information
b) it sometimes recognize grammar that is not regular
c) it sometimes fails to recognize grammar that is regular
d) all of these

Answer: (a)

29. Can a DFA simulate NFA? **[WBUT 2012]**

a) no **b) yes** **c) sometimes** **d) depends on DFA**

Answer: (b)

30. The logic of pumping lemma is a good example of **[WBUT 2013, 2019]**

a) The pigeon-hole principle
b) the divide and conquer technique
c) Recursion
d) Iteration

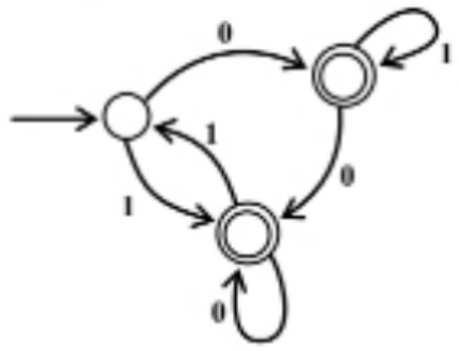
Answer: (a)

31. Which of the following strings can be obtained by the language $L = \{a^i b^{2i} \mid i \geq 1\}$ **[WBUT 2013, 2019]**

a) aaabbbbbb b) aabbb c) abbabbba d) aaaabbbabb

Answer: (a)

32. Which string is not accepted by the following FSA? **[WBUT 2013]**



a) 00111 b) 00110 c) 01010 d) 11010

Answer: (a)

33. Regular sets are closed under **[WBUT 2014, 2015]**

a) Union b) Concatenation

c) Kleene Closure d) All of the above

Answer: (d)

34. Let N be an NFA with n states and let M be the minimized DFA with m states recognizing the same language. Which of the following is NECESSARILY true? **[WBUT 2014, 2015]**

a) $m \leq 2^n$ b) $n \leq m$

c) M has one accept state d) $m = 2^n$

Answer: (a)

35. The string 1101 does not belong to the set represented by **[WBUT 2014, 2015, 2017]**

a) 110*(0+1) b) 1(0+1)*101

c) (10)*(01)*(00+11)* d) (00+(11)*01)*

Answer: (c)

36. $A = (a+b)^* a$ and $B = b(a+b)^*$ then A intersection B will be **[WBUT 2016]**

a) $(a+b)^* ab$ b) $ab(a+b)^*$ c) $a(a+b)^* b$ d) $b(a+b)^* a$

Answer: (d)

37. The string 1101 does not belong to the set represented by **[WBUT 2016]**

a) $110^*(0+1)$ b) $1(0+1)^*101$

c) $(10)^*(01)^*(00+11)^*$ d) $\left[00+(11)^*0\right]^*$

Answer: (d)

38. An automata is a/an ..... device. **[WBUT 2016]**

a) acceptor only b) acceptor / rejector

c) rejector only d) generating

Answer: (b)

39. k -level equivalence is possible between two finite automata [WBUT 2016]
a) if all equivalence above k -level exist between them
b) if all level equivalence up to $k-1$ level already exist
c) if both contain at least k number of states
d) if both contain exactly k number of states

Answer: (b)

40. In Moore machine, output is associated with [WBUT 2017]
a) present state only b) next state only
c) present state and input d) none of these

Answer: (a)

41. Regular grammar is [WBUT 2017]
a) context free grammar b) context sensitive grammar
c) non-context grammar d) none of these

Answer: (a)

42. Regular expression is accepted by [WBUT 2017]
a) Finite automata b) Mealy machine
c) Pushdown automata d) all of these

Answer: (d)

43. Pumping Lemma for Regular Expression is used to prove that [WBUT 2018]
a) Certain sets are Regular
b) Certain sets are not Regular
c) Certain Regular Grammar produce Regular Expression
d) Certain Regular Grammar does not produce Regular Expression

Answer: (b)

44. Number of vertices of a Merger graph is [WBUT 2018]
a) The number of state of the machine b) Number of compatible pairs
c) Number of states combinations d) None of the above

Answer: (a)

45. Number of vertices of a compatible graph is [WBUT 2018]
a) the number of states of the machine b) number of compatible pairs
c) number of states combinations d) None of the above

Answer: (b)

Short Answer Type Questions

1. What is a regular expression? Give a complete procedure to convert a regular expression into its equivalent NFA. Using that procedure convert $01[((10)^*+111)^*+0]^*1$ into its equivalent NFA. [WBUT 2003]

OR,

What is regular expression? **[WBUT 2013]**

Answer:

Let

i) $r_1 = (10)^*$

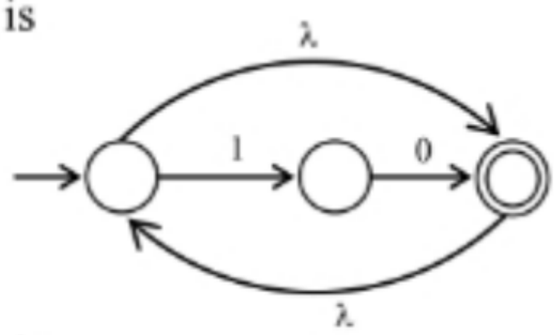
ii) $r_2 = (r_1 + 11)^*$

iii) $r_3 = (r_2 + 0)^*$

iv) $r_4 = r_3 1$

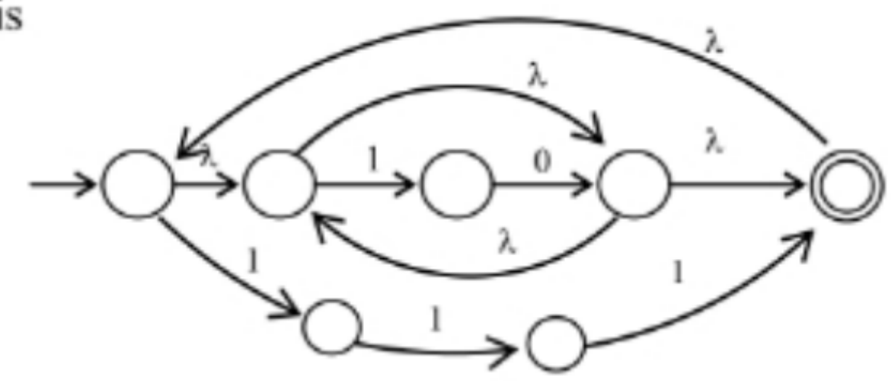
v) $r_5 = 01r_4$

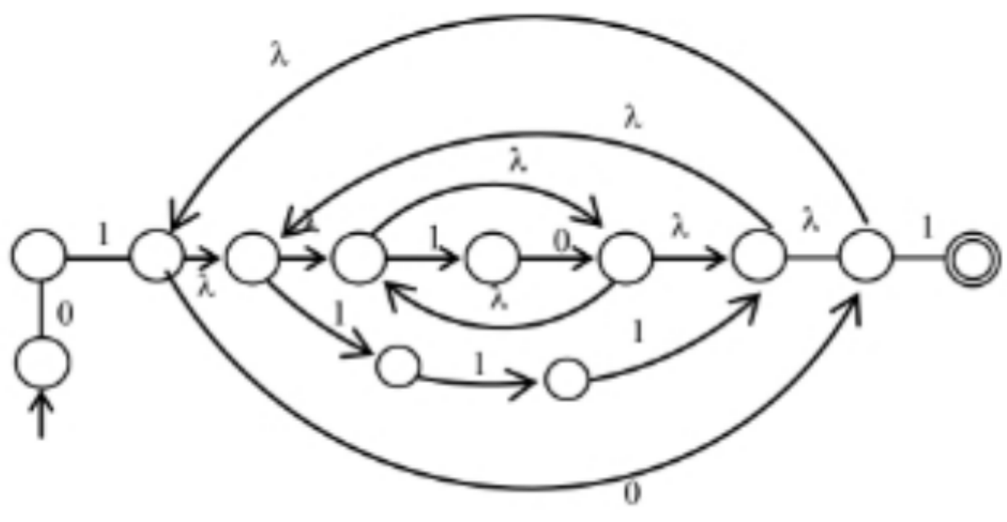
Corresponding nfa of regular expression r_1 is



Corresponding nfa of regular expression r_2 is

In this way the final nfa is





2.

PS	NS, Z		
	I_1	I_2	I_3
A	C, 0	E, 1	–
B	C, 0	E, –	–
C	B, –	C, 0	A, –
D	B, 0	C, –	E, –
E	–	E, 0	A, –

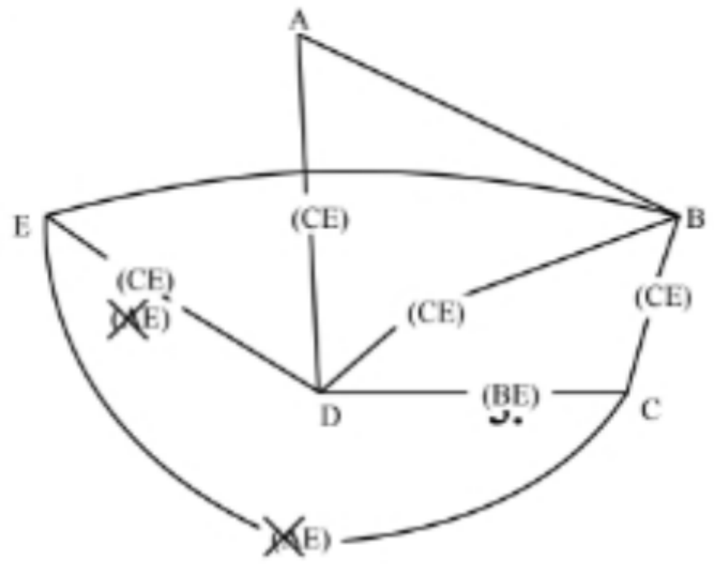
Table-1

For the incompletely specified machine shown in table-1, find a minimum state reduced machine containing the original one. [WBUT 2004, 2008, 2012]

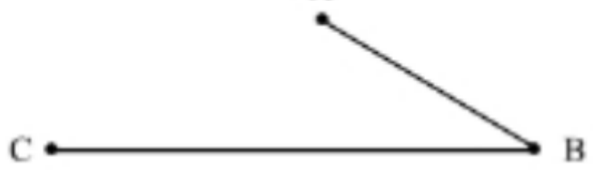
Answer:

Compatible pairs = AB, AD, BC, BD, BE, CD, CE, DE

1.



Final merger graph



Compatibility Graph

Merger table

B	✓			
C	×	~~CE~~		
D	~~CE~~	~~CE~~	BE	
E	×	✓	×	×
	A	B	C	D

2.

POPULAR PUBLICATIONS

B	✓			
C	×	CE		
D	CE	CE	BE	
E	×	✓	~~AE~~	~~CE, AE~~
	A	B	C	D

Next step merger table.

B	✓			
C	×	×		
D	×	×	BE	
E	×	✓	×	×
	A	B	C	D

Final merger table

3. State Myhill-Nerode theorem. **[WBUT 2005, 2007, 2008, 2010, 2019]**

OR,

State Myhill-Nerode theorem with the definition of equivalent relation and Invariance. **[WBUT 2013]**

Answer:

Statement of the Theorem

The Myhill-Nerode Theorem says the following three statements are equivalent:

1. The set L, a subset of Sigma star, is accepted by a DFA. (We know this means L is a regular language.)
2. L is the union of some of the equivalence classes of a right invariant (with respect to concatenation) equivalence relation of finite index.
3. Let equivalence relation RL be defined by: x RL y if and only if for all $z \in \Sigma^*$ star, xz $\in$L exactly when yz $\in$L. Then RL is of finite index.

4. Prove that the language $L=\{o^n 1^n \mid n>0\}$ is not regular. **[WBUT 2006, 2007, 2012]**

Answer:

Suppose L is regular. There exists a finite state automation m which accepts L. Suppose M has k states. Let $w = a^k b^k$. Then $|w| > k$. By the pumping Lemma w = xyz where y is not empty and $w_2 = xy^2z$ is also accepted by M. If y consists of only a's or only b's the w_2 will not have the same number of a's as b's. If y contains both a's and b's, then w_2 will have a's followed b's. In either case w_2 does not belong to L, which is contradiction. Thus L is not regular.

5. Minimize the following machine to standard form: **[WBUT 2006, 2008]**

PS	NS, z	
	X = 0	X = 1
A	E, 0	C, 0
B	C, 0	A, 0
C	B, 0	G, 0
D	G, 0	A, 0
E	F, 1	B, 0
F	E, 0	D, 0
G	D, 0	G, 0

Answer:

Let's do the partitioning process.

Partitions	Reason
$P_0 = (A\,B\,C\,D\,E\,F\,G)$	
$P_1 = (A\,B\,C\,D\,F\,G)\ (E)$	E has $o/p = 1$ on $i/p = 0$
$P_2 = (BCDG)(AF)(E)$	$(A\,F)$ goes to E on $i/p = 0$
$P_3 = (B\,D)(CG)(AF)(E)$	$(B\,D)$ goes to A on $i/p = 1$
$P_4 = (B\,D)(CG)(A)(F)(E)$	A goes to C but F goes to D on $i/p = 1$
$P_5 = (B\,D)(CG)(A)(E)(F)$	Final partition.

Let $(A) \Rightarrow \alpha$

$(BD) \Rightarrow \beta$

$(CG) \Rightarrow \gamma$

$(F) \Rightarrow \delta$

$(F) \Rightarrow \theta$

	NS Z	
P S	$X = 0$	$X = 1$
α	$\delta, 0$	$\gamma, 0$
β	$\gamma, 0$	$\alpha, 0$
γ	$\beta, 0$	$\gamma, 0$
δ	$\theta, 1$	$\beta, 0$
θ	$\delta, 0$	$\gamma, 0$

Reduced machine

6. Prove that following identity: $r(s+t) = rs + rt$. **[WBUT 2007]**

Answer:

Suppose R, S and T denote the languages for r, s, and t, respectively.
Now, L(s + t) = S U T.
Hence L(r(s + t)) = R(S U T) = RS U RT [from Set theory] = L(rs + rt)
Hence L(r(s + t)) = L(rs + rt). QED.

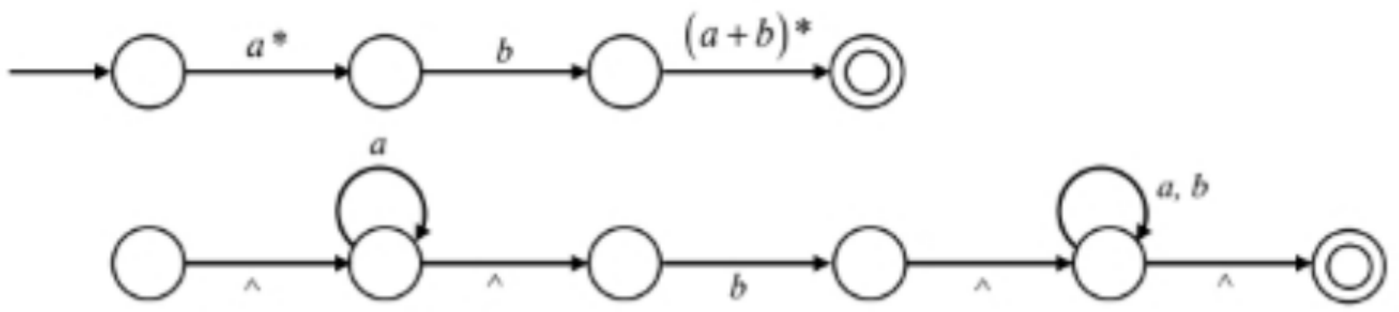
7. Construct a regular grammar *G* generating the regular set represented by

$P = a^*b(a+b)^*$ **[WBUT 2007, 2010, 2016]**

Answer:

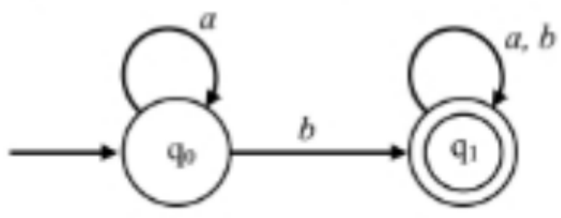
Construct a regular grammar G generally the regular set represented by $P = a + b(a+b)^*$

We construct the DFA corresponding to P using the construction is given the following figure.



DFA with ^ moves

After eliminating ^-moves, we get the DFA straight away,



DFA without ^ moves

Let $G = \{(A_0, A_1),\ (a, b),\ P,\ A_0\}$ where P is given by

$A_0 \to aA_0$ $\quad$ $A_1 \to aA_1$

$A_0 \to bA_1$ $\quad$ $A_1 \to bA_1$

$A_0 \to b$ $\quad$ $A_1 \to a$

$A_1 \to b$

G is required regular grammar.

8. Let R be an equivalence relation in $\{0\}^*$ with the following equivalence classes:

$$[\ ]_R = \{0\}^0$$

$$[0]_R = \{0\}^1$$

$$[00]_R = \{0\}^2 \cup \{0\}^3 \cup \{0\}^4 \ldots\ldots\ldots$$

Show that R is a right invariant. **[WBUT 2007, 2014, 2015]**

Answer:

If R is a right-invariant the for all x, y, z xRy => xzRyz.

Clearly, $[\ ]_R$ has the null-string, $[0]_R$ has only the string 0 and $[00]_R$ has strings 00, 000, 0000, etc.

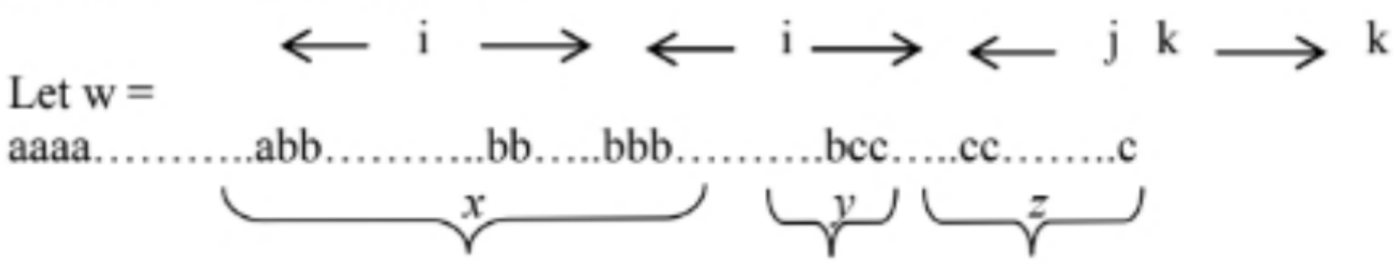
Suppose x and y are in $[\ ]_R$. Then, x = y = null. Hence xz = z and yz = z and zRz is trivially true.

Suppose x and y are in $[0]_R$. Then x = y = 0. If z is from $[\ ]_R$, then z = null, i.e., xz = x and yz = y. Hence xzRyz. If z is from $[0]_R$, then z = 0. Thus, xz = 00 and yz = 00. Hence xz = yz implying that xzRyz. Finally, if z is from $[00]_R$, z has at least two 0-s. Therefore xz as well as yz have at least three zeroes and hence both belong to $[00]_R$, i.e., xz R yz.

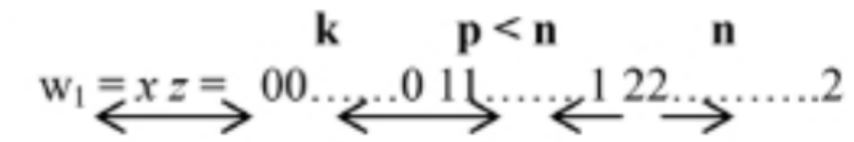
Suppose x and y are in $[00]_R$. Then both x and y have at least two 0-s. Hence irrespective of what z is, xz as well as yz will have at least two 0-s and thus both belong to $[00]_R$. Thus xzRyz.
So we see that for all cases xRy => xzRyz. Thus R is right invariant.

9. Consider the set $L=\{a^i b^j c^k \mid$ where i, j, k are integers and $i, j, k \geq 1\}$. Is L regular? Justify your answer. **[WBUT 2008, 2019]**

Answer:
Suppose L is regular
Let n be given by pumping lemma.



Be broken up as shown $|y|>0$
Hence $xy^r z \in L, r \geq 0$ by pumping lemma.



Let r = 0 has less 1's than 2's, thus leading to a contradiction.

10. What do you mean by Inverse machine? Write the definition of a lossless machine. **[WBUT 2008]**

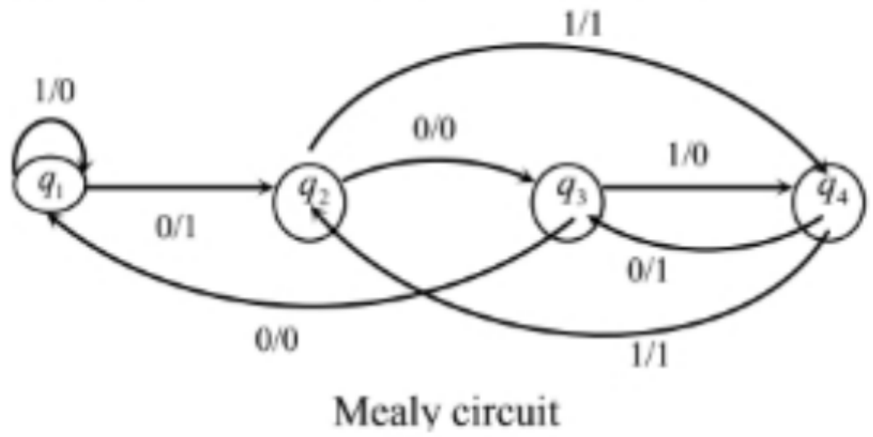
Answer:
An *inverse* M^{-1} is a machine which, when excited by the output sequence of a machine *M*, produces (as its output) the input sequence to *M*, after at most a finite delay. Evidently, a deterministic inverse can be constructed only if *M* is lossless, and it can be constructed so that it produces *M's* input sequence after just a finite delay if and only if *M* is lossless of finite order.
A machine is to be *(information) lossless* if the knowledge of the initial state, the output sequence, and the final state is sufficient to determine uniquely the input sequence.

11. Convert the Mealy Machine (given below) to a Moore Machine. **[WBUT 2008]**

Present State	Next State	i/p=0	Next State	i/p=1
	State	Output	State	Output
Q_1	Q_2	1	Q_1	0
Q_2	Q_3	0	Q_4	1
Q_3	Q_1	0	Q_4	0
Q_4	Q_3	1	Q_2	1

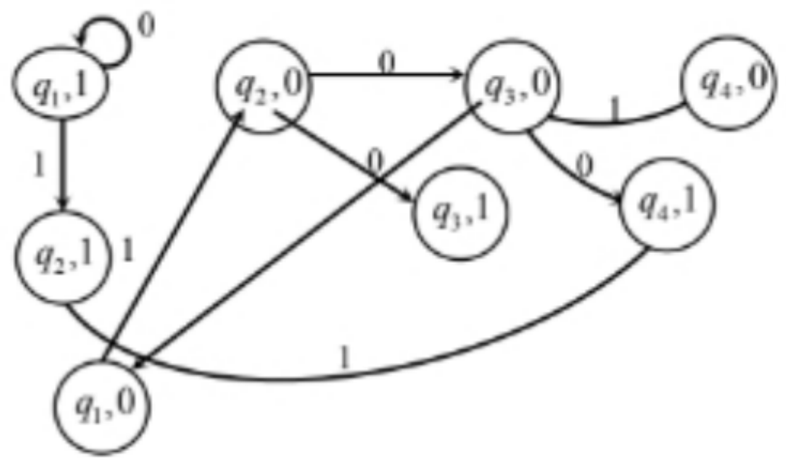
Answer:

The Mealy machine constructed form transition diagram is



Mealy circuit

The equivalent Moore circuit is given by



12. Construct a Mealy machine which is equivalent to the Moore machine given below: **[WBUT 2009]**

PS	*NS*		*z*
	$x=0$	$x=1$	
q_0	q_1	q_2	**1**
q_1	q_3	q_2	**0**
q_2	q_2	q_1	**1**
q_3	q_0	q_3	**1**

Answer:

	NS, *z*	
	$x=0$	$x=1$
q_0	$q_1, 0$	$q_2, 1$
q_1	$q_3, 1$	$q_2, 1$
q_2	$q_2, 1$	$q_1, 0$
q_3	$q_0, 1$	$q_3, 1$

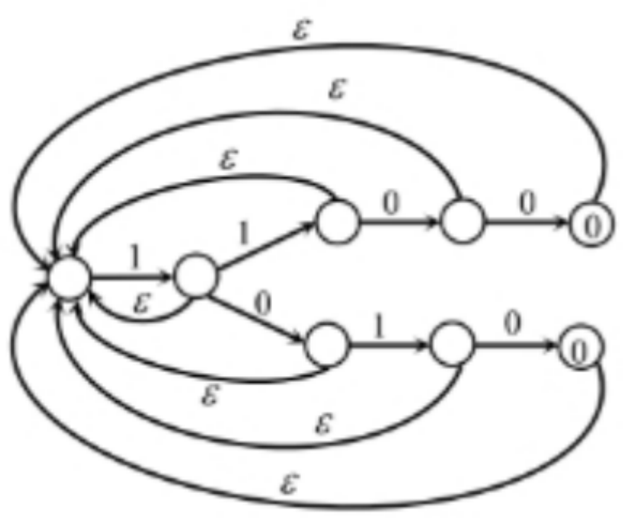
13. State the difference between DFA and NFA. **[WBUT 2009, 2010]**

Answer:

In a DFA, these can be no ε - transitions. Also, for a given state q and a symbol $a \in \sum$, there can be at most one entry. An NFA does not have any of the above restrictions.

14. Design an NFA which accepts set of all binary strings containing 1100 or 1010 as substrings. **[WBUT 2009, 2010, 2016]**

Answer:



15. What is the basic difference between Mealy machine and Moore machine? **[WBUT 2009]**

OR,

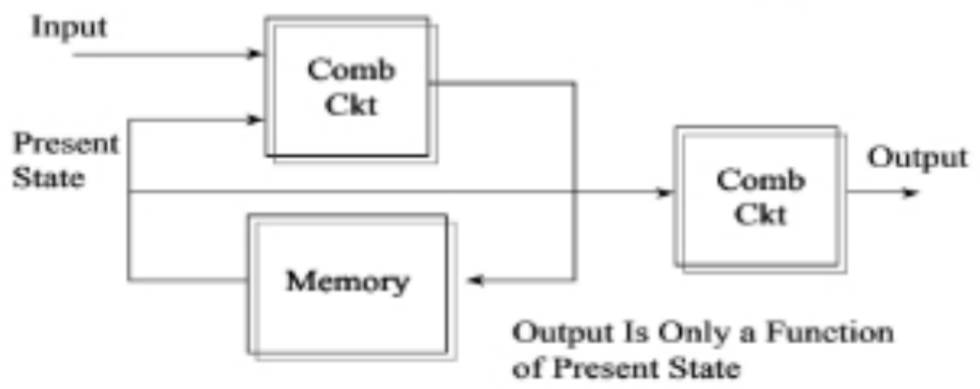
Define and compare Moore and Mealy machines. **[WBUT 2015]**

Answer:

Moore Machine

The limitation of FA is that its output is limited to a binary signal "accept/don't accept". Models in which the output is chosen from some other alphabet and in which the output is associated with the state is called Moore machine.

A Moore machine is a six-tuple $(Q, \sum, \Delta, \delta, \lambda, q_0)$ where Q, $\sum$, δ and q_0 are as in a DFA. Δ is the output alphabet and λ is a mapping from Q to Δ giving the output associated with each state. Any Moore machine gives output $\lambda(q_0)$ in response to input $\in$. The DFA may be viewed as a special case of a Moore machine, where the output alphabet is {0, 1} and state q is "accepting" if and only if $\lambda(q) = 1$.



Block diagram of Moore Machine

Mealy Machine

A Mealy machine is a sequential machine in which output is associated with each state transition.

The output of the vending machine example, presented in class, is that of a Mealy machine.

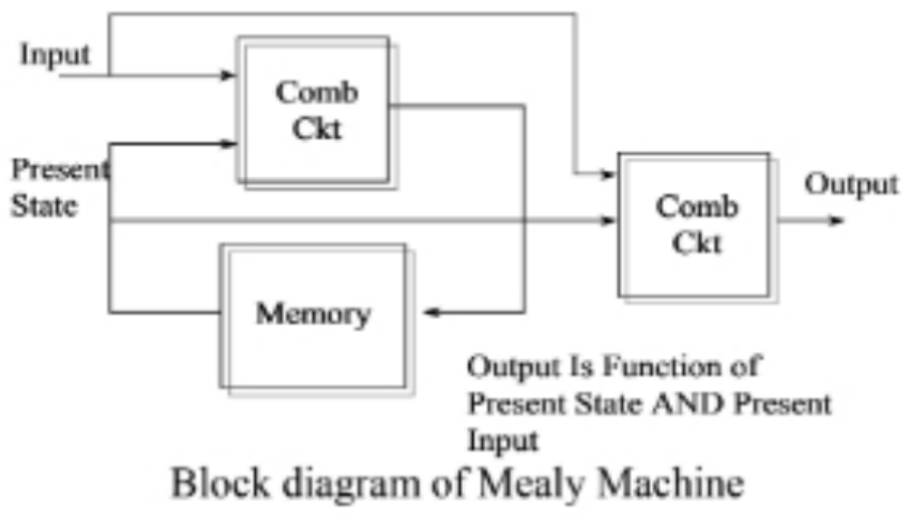
A Mealy machine is a six-tuple $M = (Q,\Sigma, \Delta,\lambda,q_0)$, where Q is a finite non-empty set of states, Σ is the input alphabet, Δ is the output alphabet, $\lambda : Q\times\Sigma \rightarrow Q$ is the state transition function, and $\lambda: Q \times\Sigma \rightarrow \Delta$ is the output function.

The λ function gives the output symbol associated with the transition from state q on input symbol α, i.e.,

$\lambda(q, \alpha) = z \in \Delta$

Notice the six-tuple does not have a set of final states. That is because sequential machines do not have final states.

The length of the output string is equal to the length of the input string. The state transition diagram for a Mealy machine M is almost identical to the diagram for finite automata; the output symbol is denoted on the edge by appending to the input symbol, a slash ("/") followed by the output symbol. Sequential machines are used to design sequential circuits.



Block diagram of Mealy Machine

OR Part also:

Moore vs. Mealy Machine

Moore Machine: Output only depends on present state

Mealy Machine: Output depends on both the present state and the present input.

16. In response to an unknown input sequence, the machine given below produces the output sequence 1110000010. Find the input sequence to the machine if it is known that its initial state is *A* and final state is *F*. **[WBUT 2009]**

PS	*NS, z*	
	$x=0$	$x=1$
A	B, 1	C, 0
B	D, 1	B, 1
C	E, 1	B, 0
D	A, 0	E, 0
E	F, 0	D, 1
F	D, 0	A, 1

Answer:
This problem is best solved by tracing backwards from state F. We see that the only F, 0 entry is for E and $x = 0$. Hence the second-last state is E. Proceeding similarly. We get

Output: 1 1 1 0 0 0 0 0 1 0
Possible states F A B D E F D A C E F
A B E
B C

But we know that the first state is A and since the first output is 1, the next state is B. Also, since the next output is 1, the next state is B or D. But we have found that this state is B or E. Hence, the state is B. So, the state transition are:

A → B → B → D → E → F → D → A → C → E → F

Hence, the input string is: 0101000110

17. Is the following machine information lossless? If yes, find the order of losslessness. **[WBUT 2009]**

PS	NS, z	
	$x = 0$	$x = 1$
A	A, 0	B, 0
B	C, 0	D, 0
C	D, 1	C, 1
D	B, 1	A, 1

Answer:
The first step of this testing procedure is to check each row of the transition table, whether there is two identical next state entries associated with the same output symbol. From the above table it is clear that, there is no identical next state entries.
Now we have to construct the output successor table.

Testing table for Machine-M

PS	z = 0	z = 1
A	(AB)	—
B	(CD)	—
C	—	(CD)
D	—	(AB)
AB	(AB) (CD)	—
CD	—	(AB) (CD)
AC	—	—
AD	—	—
BC	—	—
BD	—	—

The lower part of the testing table is constructed from the compatible pair of states of the upper part of the table. In the lower part of the table all the implied pairs have been taken as a row heading.

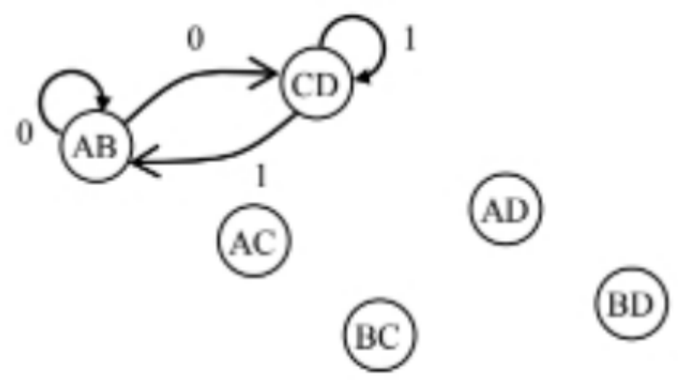


Fig: Testing graph for machine M

From the lower part of the testing table it is clear that, there is no compatible pair with repeated entries. This is necessary and sufficient for a machine to be lossless.
So clearly the given machine is lossless.
In the above testing graph the length of the largest path is $l = 2$.
Hence the order of losslessness, $\mu = l + 2 = 2 + 2 = 4$

18. Show that $L = \{a^p \mid p \text{ is prime}\}$ is not regular. **[WBUT 2009]**

Answer:
L = {a^P : P is Prime}
Let L be regular. Let "adversary" choose n.
Let z = a^P, where P > n. Clearly |z| >= n.
Let adversary choose a decomposition uvw satisfying the conditions of pumping lemma.
Let |u| = p, |v| = q and |w| = r
So we have:
q > 0
p + q < n
p + q + r = P

By pumping lemma, uv^iw is in L. In particular, let us choose i = q^p and let y = $uv^{q^p}w$.
By pumping lemma, y should be in L.
Thus $|y| = p + q^* q^p + r$
By Fermat's theorem, q^p= s * P + q for some integer s.
Hence:
|y| = p + s * P + q + r = s * P + (p + q + r) = (s + 1) * P
Since (s + 1) * P is not a prime, y is not in L, leading to a contradiction. So L is not regular.

19. a) State the pumping lemma for regular language. **[WBUT 2010]**
b) Using pumping lemma prove that the set $L = \{0^i 1^i \mid i \geq 1\}$ is not regular.

Answer:
a) Formal statement of the Pumping Lemma
Let L be a infinite Regular Language. Then there exists some positive integer n such that any z ∈ L with $|z| \geq n$

Can be decomposed as

$z = uvw$

with $|uv| \leq n$,

and $|v| \geq 1$,

such that $z_i = uv^i w$,

is also in L for all i=0,1,2,….

The two commonest ways to use the Pumping Lemma to prove a language is NOT regular are:

- show that there is no possible n for the (there exists n), this is usually accomplished by showing a contradiction such as $(n+1)(n+1) < n*n+n$
- show there is no way to partition z into u, v and w such that uv^iw is in L

b) $L = \{a^n b^n \; n > 0\}$

Let L be regular. Let "adversary" choose n.

We choose $z = a^n b^n$. Obviously, $|z| >= n$

Now, let adversary choose decomposition z = uvw such that |uv| <= n and |v| > 0.

We note that whatever be the decomposition, the string uv consists only of 0-s and v has at least one 0. Therefore, for i > 1, uv^i has more 0-s than uv and hence the string uv^iw has more 0-s at the beginning than there are 1-s at the end, allowing us to conclude that uv^iw is not in L. Hence L is not regular. QED.

Note: Similar logic can be applied to prove the languages

$L = \{a^n b^k \; n > 0, k > n\}$ non-regular.

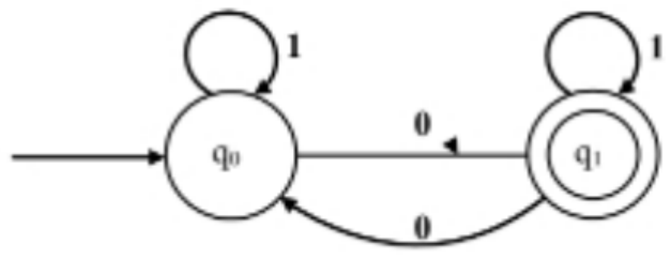
20. Draw the transition diagram of a finite state automaton that accepts all strings over {0, 1}

a) having odd number of 0's

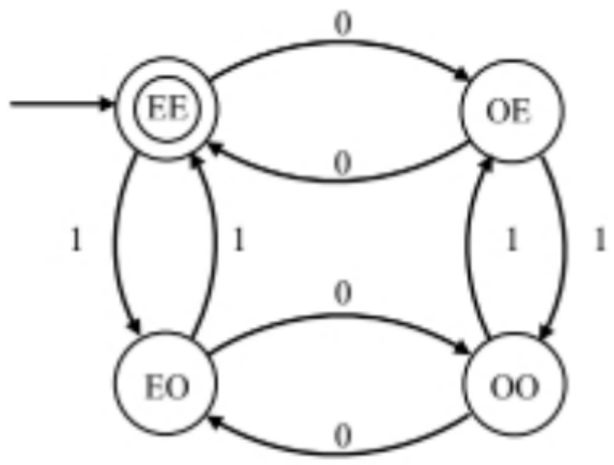
b) having even number of 0's and even number of 1's. **[WBUT 2010]**

Answer:

a) $(1*01*)* + 1*$



b) $(0*1*0*!*)*$

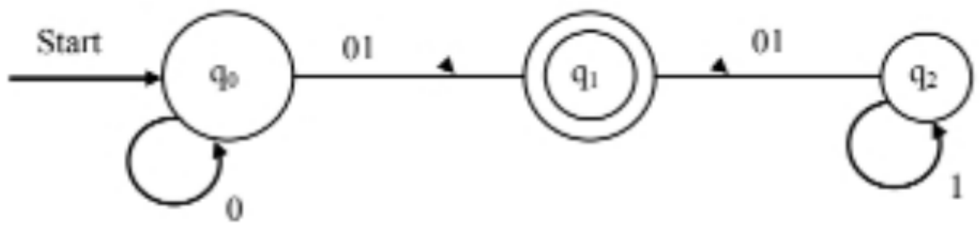


21. State the difference between DFA and NFA. **[WBUT 2010, 2018]**

Answer:

1. The transition function for nfa ie delta is multi valued where as for dfa it is single valued.
2. Checking membership is easy with dfa where as it is difficult for nfa
3. Construction of nfa is very easy where as for dfa it is difficult
4. Space required for dfa is more where for nfa it is less
5. Backtracking is allowed in dfa, but it is not possible in every casi in nfa.
6. For every input and output we can constuct dfa machine, but it is not possible to construct an nfa machine for every input and output.
7. There is only 1 final state in nfa but there can be more then 1 final state in dfa.

22. Convert the following NFA to DFA. **[WBUT 2010]**



Answer:

NFA

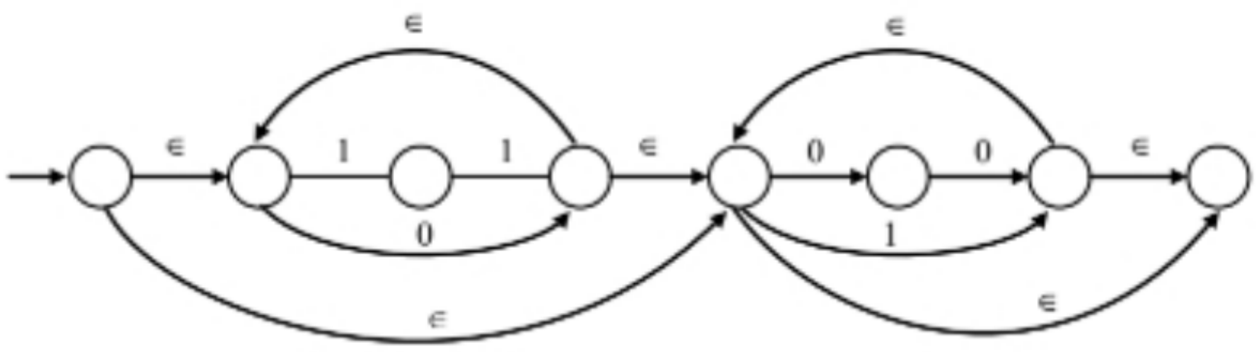
State/∈	0	1
$\rightarrow q_0$	q_0, q_1	q_1
(q_1)	q_2	q_2
q_2	–	q_2

DFA

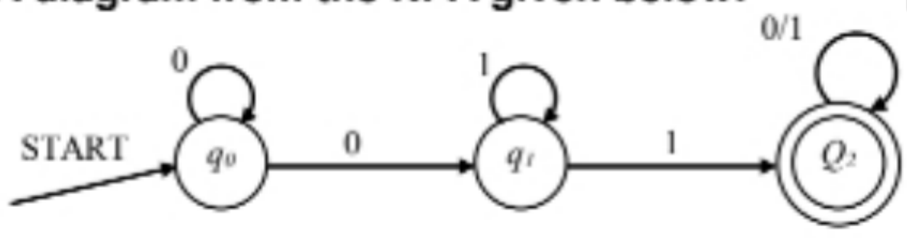
Q	Q_o	Q_1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$[q_1, q_2]$	$[q_2]$	$[q_2]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$[q_2]$	ϕ	$[q_2]$
$[q_1]$	$[q_2]$	$[q_2]$
ϕ	ϕ	ϕ

23. Construct a NFA with ε or λ transition for $r=(11+0)^*(00+1)^*$. [WBUT 2011, 2012]

Answer:

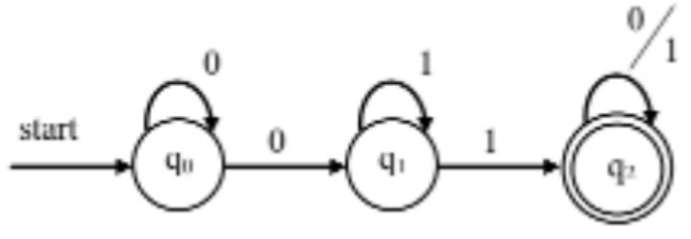


24. Construct a DFA diagram from the NFA given below: [WBUT 2011, 2012]



Answer:

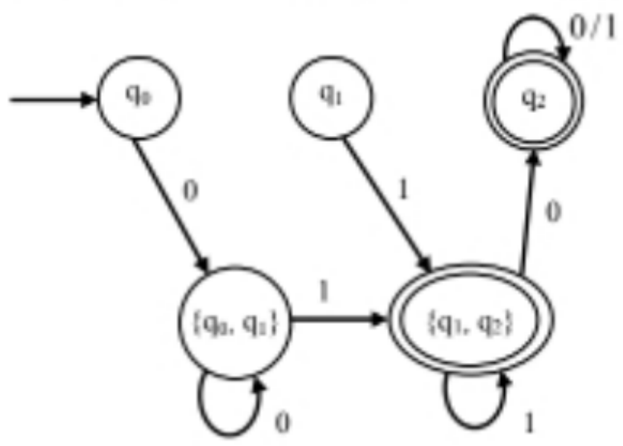
The NFA diagram:



According to the above diagram, the transition table will be formed as:

Present state	i/p 0	i/p 1
→ q_0	$\{q_0, q_1\}$	$\{\phi\}$
q_1	$\{\phi\}$	$\{q_1, q_2\}$
(*) q_2	q_2	q_2
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	q_2	$\{q_1, q_2\}$

Now, according to the above table the DFA diagram will be,



The DFA Diagram

25. What are Kleene Closure and Positive Closure? Give example for both.
[WBUT 2011, 2012]

OR,

What is Klenne's star? Give example. What is positive closure? Give example.
[WBUT 2016]

Answer:

For a Regular expression R, Kleene closure, denoted by R^*, is defined to be the expression obtained by concatenating *zero* or more R-s. Thus, $R^* = \varepsilon \mid R \mid RR \mid RRR \mid \ldots$, where the vertical bar means disjunction. A Positive closure, denoted by $R+$, is defined to be the expression on the other hand is obtained by concatenating *one* or more R-s. Thus, $R+ = \in R \mid RR \mid RRR \mid \ldots$

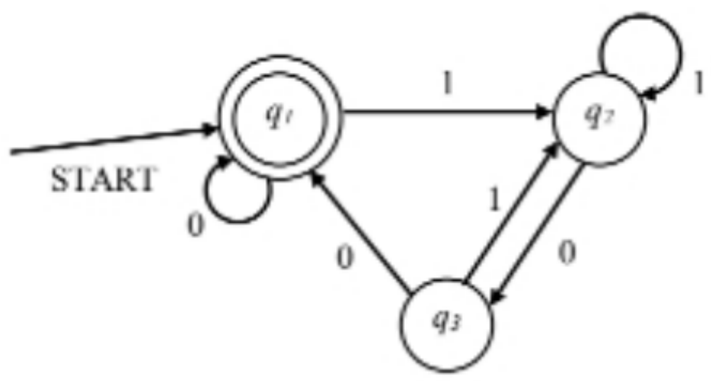
Example:

Suppose $R = a \mid bb$ Then,

$R^* = \varepsilon \mid a \mid bb \mid aa \mid abb \mid bba \mid bbbb \mid$

$R+ = a \mid bb \mid aa \mid abb \mid bba \mid bbbb \mid$

26. Give the Regular Expression for the DFA using Arden Theorem.
[WBUT 2011, 2012, 2016]



Answer:

The state equations are

$q_1 = \varepsilon + q_1 0 + q_3 0$ ....(1)

$q_2 = q_1 1 + q_2 1 + q_3 1$ ....(2)

$q_3 = q_2 0$ ....(3)

Substituting (3) in (2) we get:

$q_2 = q_1 1 + q_2 (1 + 01)$ ....(4)

We know that if $R = Q + RP$, then $R = QP^*$

Hence, from (4),

$q_2 = q_1 1 (1 + 01)^*$ ....(5)

Substituting (3) and (5) in (1)

We get:

$q_1 = \varepsilon + q_1 0 + q_2 00$

$= \varepsilon + q_1 (0 + 1(1 + 01)^* 00)$

$$\text{or } q_1 = \varepsilon\left(0+1(1+01)^*00\right)^*$$
$$= \left(0+1(1+01)^*00\right)^*$$

Which is the RE for the given transition diagram.

27. Using Pumping Lemma show that $L=\{a^n b^n : n \geq 0\}$ is not regular.

[WBUT 2011, 2012]

Answer:

Let L be regular. Let "adversary" choose n.

We choose $z = a^n b^n$. Obviously, $|z| >= n$

Now, let adversary choose decomposition $z = uvw$ such that $|uv| <= n$ and $|v| > 0$.

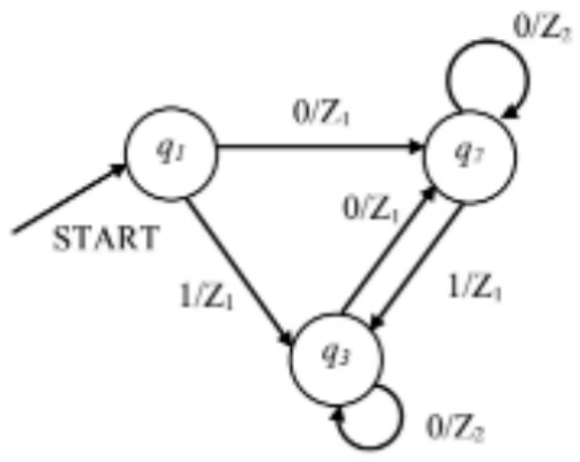
We note that whatever be the decomposition, the string uv consists only of 0-s and v has al least one 0. Therefore, for $i > 1$, uv^i has more 0-s than uv and hence the string uv^iw has more 0-s at the beginning than there are 1-s at the end, allowing us to conclude that uv^iw is not in L. Hence L is not regular. QED.

Note: Similar logic can be applied to prove the languages

$L = \{a^n b^k \;\; n > 0, k > n\}$ non-regular.

Suppose L is regular. There exists a finite state automation m which accepts L. Suppose M has k states. Let $w = a^k b^k$. Then $|w| > k$. By the pumping Lemma $w = xyz$ where y is not empty and $w_2 = xy^2z$ is also accepted by M. If y consists of only a's or only b's the w_2 will not have the same number of a's as b's. If y contains both a's and b's, then w_2 will have a's followed b's. In either case w_2 does not belong to L, which is contradiction. Thus L is not regular.

28. Convert Mealy Machine to Moore Machine. **[WBUT 2011, 2012, 2016]**



Answer:

According to the above diagram the Melay machine is transition diagram will be,

PS	NS, z	
	i/p = 0	i/p = 1
q_1	$\{q_2, z_1\}$	$\{q_3, z_1\}$
q_2	$\{q_2, z_2\}$	$\{q_3, z_1\}$
q_3	$\{q_3, z_2\}$	$\{q_2, z_1\}$

⇓

PS	NS, z	
	i/p = 0	i/p = 1
q_1	$\{q_2, z_1\}$	$\{q_3, z_1\}$
q_{20}	$\{q_{20}, z_2\}$	$\{q_3, z_1\}$
q_{21}	$\{q_{20}, z_2\}$	$\{q_3, z_1\}$
q_{30}	$\{q_{30}, z_2\}$	$\{q_{21}, z_1\}$
q_{31}	$\{q_{30}, z_2\}$	$\{q_{21}, z_1\}$

The corresponding Moore Machine will be,

Present State	NS		Output
	i/p = 0	i/p = 1	
q_1	q_2	q_3	z_1
q_{20}	q_2	q_3	z_2
q_{21}	q_2	q_3	z_1
q_{30}	q_3	q_2	z_2
q_{31}	q_3	q_2	z_1

29. Is the following machine information lossless? If yes, find the order of losslessness. **[WBUT 2011]**

	NS, z	
	$X = 0$	$X = 1$
A	*A*, 0	*B*, 0
B	*C*, 0	*D*, 0
C	*D*, 1	*C*, 1
D	*B*, 1	*A*, 1

Answer:

Test of information losslessness

PS	NS, t	
	x = 0	x = 1
A	A, 0	B, 0
B	C, 0	D, 0
C	D, 1	C, 1
D	B, 1	A, 1

The first step of this testing procedure is to check each row of the transition table, whether there is two identical next state entries associated with the same output symbol.

From the above table it is clear that, there is no identical next state entries.

Now we have to construct the output successor table.

Testing table for Machine-M

PS	z = 0	z = 1
A	(AB)	—
B	(CD)	—
C	—	(CD)
D	—	(AB)
AB	(AB) (CD)	—
CD	—	(AB) (CD)
AC	—	—
AD	—	—
BC	—	—
BD	—	—

The lower part of the testing table is constructed from the compatible pair of states of the upper part of the table. In the lower part of the table all the implied pairs have been taken as a row heading.

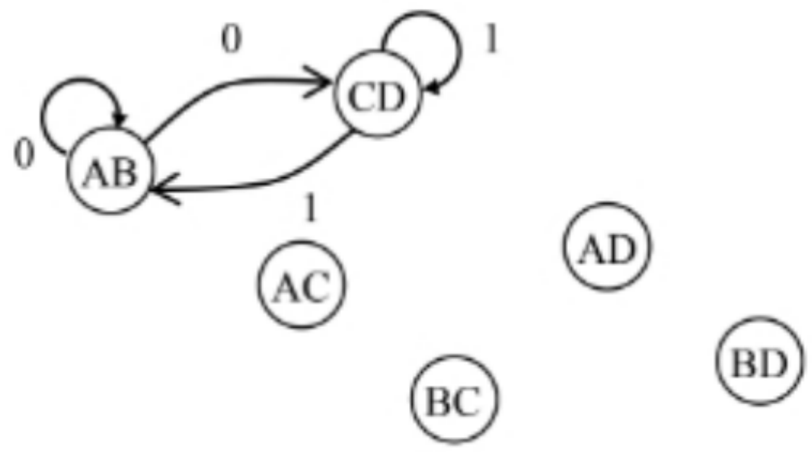


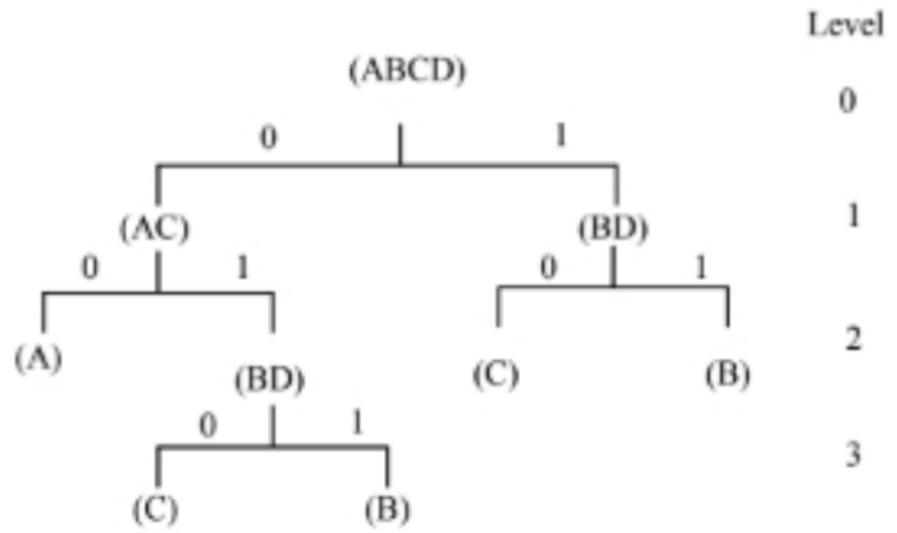
Fig.: Testing graph for machine M

From the lower part of the testing table it is clear that, there is no compatible pair with repeated entries. This is necessary and sufficient for a machine to be lossless.
So clearly the given machine is lossless.
In the above testing graph the length of the largest path is $l = 2$.
Hence the order of losslessness, $\mu = l + 2$

30. Test whether the following machine is definite or not **[WBUT 2011]**
i) by using synchronization tree
ii) by using repeated derivation of contracted table
iii) if the machine is definite,
what is the order of definiteness? Justify.

Present State	Next State	
	a = 0	a = 1
A	A	B
B	C	B
C	A	D
D	C	B

Answer:



It is clear that the tree is terminated by rule 2. So machine is definite machine.
Here k = 3
Therefore order of the definiteness is 3.

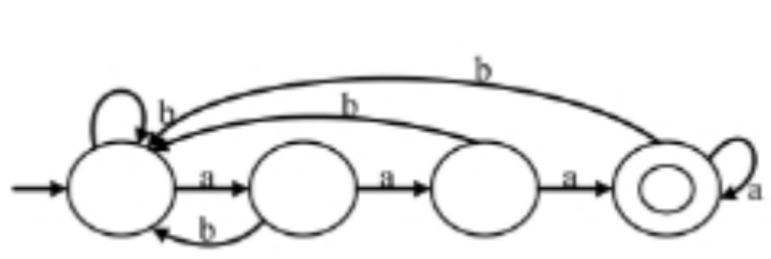
31. a) Give DFA which reads strings from $\{a,b\}$ and with aaa.

b) Construct a DFA equivalent to $M=\{\{q_0,q\},\{0,1\},\delta q_0,\{q_0\}\},\delta$ is given by the state table. **[WBUT 2012]**

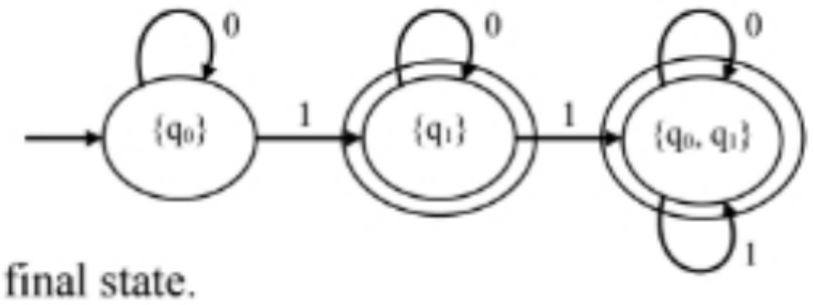
State/	0	1
q0	q 0	q 1
q1	q 1	q 0 , q 1

Answer:

a)



b)

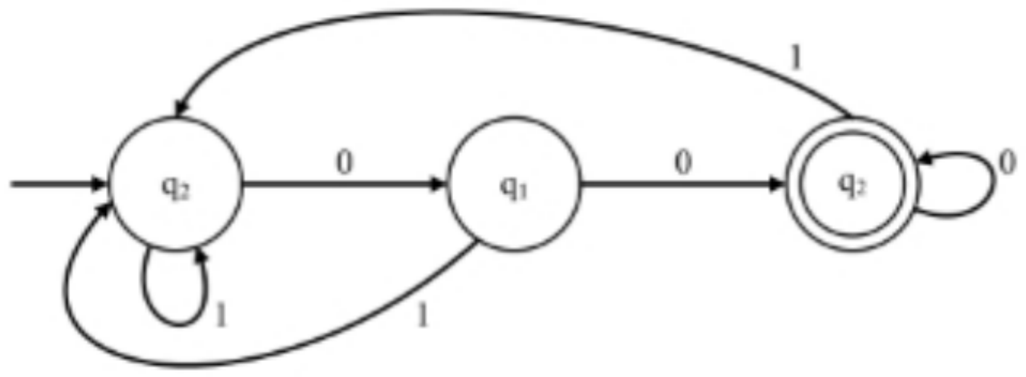


Assuming q_1 to be the final state.

32. Design a Finite automata that accepts set of strings that every string ends with 00 over alphabet {0, 1}. **[WBUT 2013]**

Answer:

FA that accepts set of strings that every string ends with 00 over alphabet $\{0,1\}$



33. What will be regular expression over the alphabet {a, b}, for the language $L=\{a^n b^m : n>=4, m<=3\}$? **[WBUT 2013]**

Answer:

The regular expression over the alphabet {a, b}, for the language $L=\{a^n b^m : n>=4, m<=3\}$ is as follows : Separate into cases m = 0, 1, 2, 3. Generate 4 or more a's, followed by the requisite number of b's.

So, the regular expression for the following language is $aaaaa^*(\lambda + b + bb + bbb)$.

34. Design a two input two output sequence detector which generates an output '1' every time the sequence 1001 is detected. And for all other cases output '0' is generated. Overlapping sequences are also counted.
[Denote State Graph, State Table and perform State assignment] **[WBUT 2014, 2015]**

Answer:

Before designing this circuit some clarifications regarding sequence detector is needed. Let's assign the input string as 1001001.

We have to design the circuit is such a way that it will take one input at a time. The input can be either '0' or '1' (two types of input). The output will be also two types, either '0' or '1'. The circuit can store (memorize) the input string up to four clock pulses (t_{i-3} to t_i).

If the input string is placed according to clock pulses, the output is as follows:

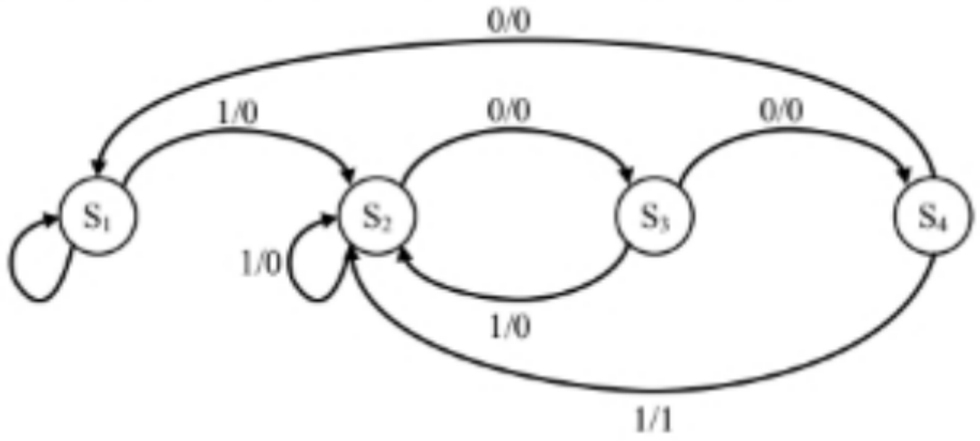
	t_1	t_2	t_3	t_4	t_5	t_6	t_7
I/P	1	0	0	1	0	0	1
O/P	0	0	0	1	0	0	1

The first input at t_1 is 1 and as there is no input from t_{i-3} to t_i the input sequence does not equal to 1001. So the output will be 0. Similar cases occur for the inputs upto t_3.

However at time t_4 the input from t_{i-3} to t_i becomes 1001, hence the output '1' is produced at time t_4. At time t_5 and t_6 the input string from t_{i-3} to t_i are 0010 and 0100, respectively. Therefore, the output '0' is produced. At t_7 clock pulse the input string is 1001, hence output '1' is produced. As the output '1' at t_4 is overlapped from t_1 to t_4 and from t_4 to t_7, this is called overlapping condition.

1 0 0 1 0 0 1

In this case the state diagram is to be drawn first according the following process:



In state S_1, input may be either '0' or '1'. If given input '0', there is no chance to get 1001. Hence it loops on S_1 with output '0'. If the input is '1', there is a chance to get 1001 then machine moves to S_2 producing output '0'.

In S_2 again the input may be either '0' or '1'. If it is '1', the input becomes 11. There is no chance to get 1001 by taking previous inputs. But again there is a chance to get 1001 by considering the given input '1'. Hence it will be in state S_2. (If it goes to S_1 then there will be loss of clock pulse, i.e., from S_1 by taking input '1', again it has to come to S_2, i.e., one extra input, i.e., one clock pulse is needed and hence the output will not be in right pattern). If the input is '0', the input becomes 10 – by considering the previous input and there is chance to get 1001, so it will come to S_3.

In S_3 if it gets input '0', the input becomes 100 by considering the previous input and it has a chance to get 1001, so it can shift to S_4. But if it gets '0', it has no chance to get 1001 considering the previous input, but there is a chance to get 1001 by considering the given input '1'. So it will shift to S_2 as we know by getting '1' in S_1 the machine comes to S_2.

In S_4 if it gets '0', the input will become 1000, but it does not match with 1001. Therefore, it has to start from the beginning i.e., S_1. As overlapping condition is accepted, hence from the last '1' of 1001 if it gets 001 only, it will give an output '1'. Therefore, it will go to S_2.

State table:

A state table can easily be constructed from the above state diagram.

PS	NS, O/P	
	X	0
S_1	$S_1, 0$	$S_2, 0$
S_2	$S_3, 0$	$S_2, 0$
S_3	$S_4, 0$	$S_2, 0$
S_4	$S_1, 0$	$S_2, 1$

State assignment:

The states must be assigned to some binary numbers to make a digital circuit. This is called state assignment. As the number of states is four, only two digit number is sufficient to represent the four states $\left(2^2 = 4\right)$.

Let's assign

S_1 to 00

S_2 to 01,

S_3 to 11,

S_4 to 10.

After doing this state assignment the state table becomes.

PS(y_1y_2)	NS(Y_1Y_2)		O/P(z)	
	X 0	1	0	1
00	00	01	0	0
01	11	01	0	0
11	10	01	0	0
10	00	01	0	1

The digital function can easily be derived from this state assignment table:

$Y_1 = X'y_2$

$Y_2 = X + y_1'y_2$

$z = Xy_1y_2'$

Y_1		
X / y_1y_2	0	1
00	0	0
01	1	0
11	1	0
10	0	0

Y_2		
X / y_1y_2	0	1
00	0	1
01	1	1
11	0	1
10	0	1

z		
X / y_1y_2	0	1
00	0	0
01	0	0
11	0	0
10	0	1

35. Define NFA. Construct equivalent DFA from the given NFA.

Present State	Next State	
	0	1
→ q0	q0, q1	q2
q1	q2	q1
(q2)	q1	q2

[WBUT 2014, 2015, 2017, 2019]

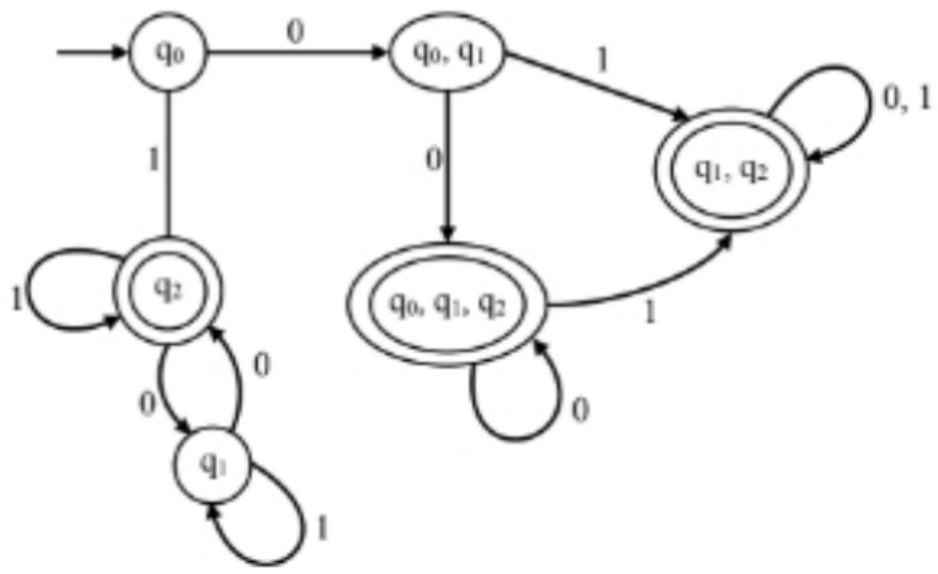
Answer:

In automata theory, a non-deterministic finite automaton (NFA), or non-deterministic finite state machine, is a finite state machine that (1) does not require input symbols for state transitions and (2) is capable of transitioning to zero or two or more states for a given start state and input symbol. This distinguishes it from a deterministic finite automaton (DFA), in which all transitions are uniquely determined and in which an input symbol is required for all state transitions. Although NFA and DFA have distinct definitions, all NFAs can be translated to equivalent DFAs using the subset construction

algorithm, i.e., constructed DFAs and their corresponding NFAs recognize the same formal language. Like DFAs, NFAs only recognize regular languages.

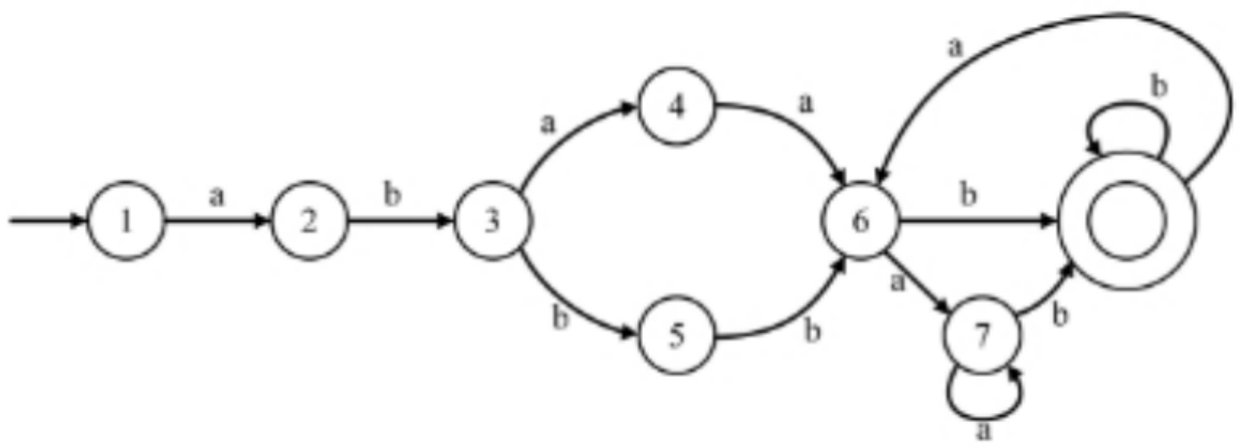
State table

	0	1
q_0	(q_0, q_1)	(q_2)
(q_0, q_1)	(q_0, q_1, q_2)	(q_1, q_2)
(q_0, q_1, q_2)	(q_0, q_1, q_2)	(q_1, q_2)
(q_1, q_2)	(q_1, q_2)	(q_1, q_2)
q_2	q_1	q_2
q_1	q_2	q_1



36. Construct Finite Automata equivalent to the Regular Expression. L=ab(aa+bb)(a+b)*b. **[WBUT 2014]**

Answer:



37. Minimize the machine using equivalent partitioning **[WBUT 2015]**

Present state	Next state,	output
	X = 0	X = 1
A	E, 0	D, 1
B	F, 0	D, 0
C	E, 0	B, 1
D	F, 0	B, 0

E	**C, 0**	**F, 1**
F	**B, 0**	**C, 0**

Answer:

$P_0 = (A,B,C,D,E,F)$
$P_1 = (A,C,E)\ (B,D,F)$
$P_2 = (A,C,E)\ (B,D)\ (F)$
$P_3 = (A,C)\ (E)\ (B,D)\ (F)$
Say: $(AC) \rightarrow \alpha$, $E \rightarrow \beta$, $(BD) \rightarrow \lambda$, $F \rightarrow \delta$
The standard form of the reduced machine is:

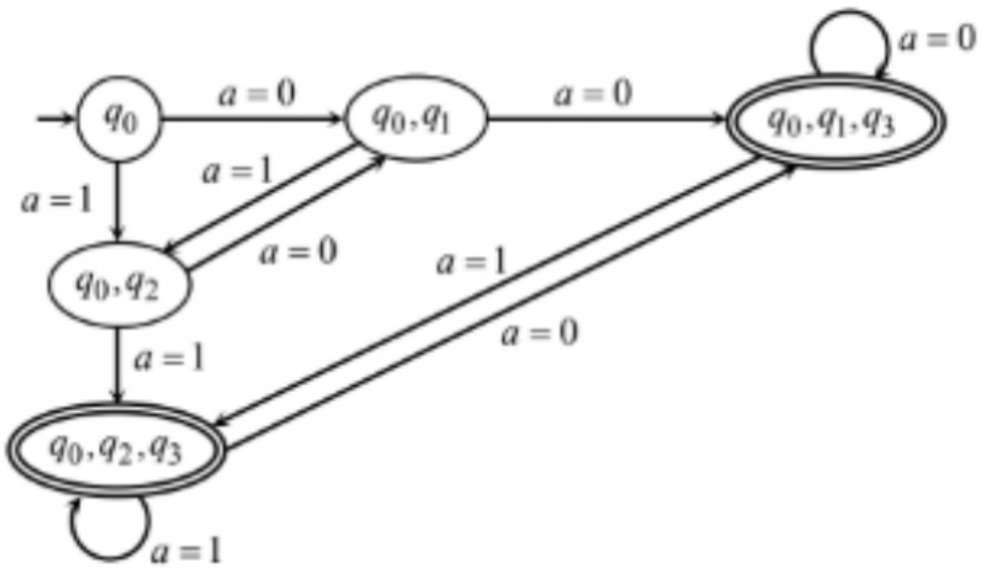
PS	**NS**	
	x = 0	**x = 1**
α	β,0	λ ,1
β	α,0	δ,1
λ	δ,0	λ ,1
δ	λ ,0	α,1

38. Convert the following NFA into an equivalent DFA. **[WBUT 2016]**

PS	*NS*	
	$a=0$	$a=1$
$\rightarrow q_0$	q_0 , q_1	q_0 , q_2
q_1	q_3	–
q_2	–	q_3
(q_3)	q_3	q_3

Answer:

PS	*NS*	
	$a=0$	$a=1$
$\rightarrow q_0$	(q_0, q_1)	(q_0, q_2)
(q_0, q_1)	(q_0, q_1, q_3)	(q_0, q_2)
(q_0, q_2)	(q_0, q_1)	(q_0, q_2, q_3)
(q_0, q_1, q_3)	(q_0, q_1, q_3)	(q_0, q_2, q_3)
(q_0, q_2, d_3)	(q_0, q_1, q_3)	(q_0, q_2, q_3)



39. Design a two-input two-output sequence detector which generates an output '1' every time the sequence 1010 is detected. And for all other cases output '0' is generated. Overlapping sequences are also counted. Draw only state table and state diagram. **[WBUT 2016]**

Answer:

The input string 1010100 is placed according to the clock pulses, and it looks like the following

	t_1	t_2	t_3	t_4	t_5	t_6	t_7
I/P	1	0	1	0	1	0	0
O/P	0	0	0	1	0	1	0

Fig: Overlapping Sequence

And the output becomes as given earlier. Overlapping portion is 10, as shown in above figure. The state diagram is in following:

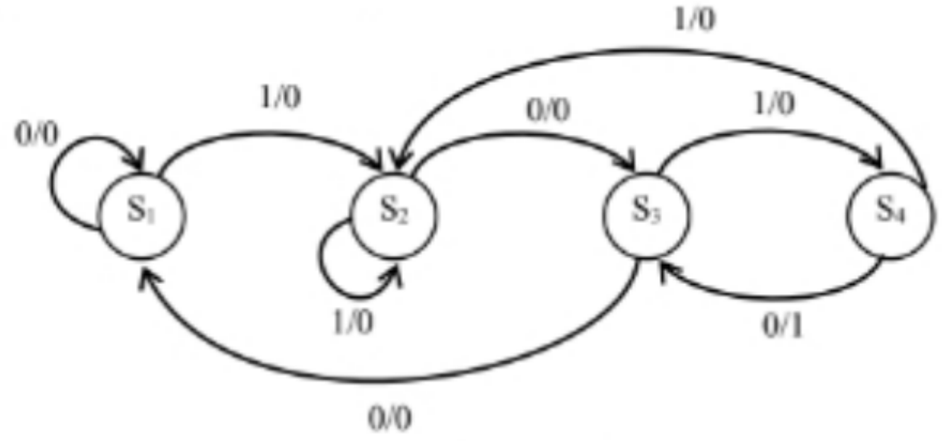


Fig: State diagram

State Table: From the previous state diagram, a state table as follows can easily be constructed.

	Next State, O/P	
Present State	X=0	=1
S_1	S_1, 0	S_2, 0
S_2	S_3, 0	S_2, 0
S_3	S_1, 0	S_4, 0
S_4	S_3, 0	S_2, 0

40. Find the string which is applied on state 'D' producing output string 10011110 and final state 'D' for the machine given below:

PS	$NS, o/p$	
	$x=0$	$x=1$
A	$A, 1$	$C, 1$
B	$E, 0$	$B, 1$
C	$D, 0$	$A, 0$
D	$C, 0$	$B, 0$
E	$B, 1$	$A, 0$

[WBUT 2016]

Answer:

First we need to prove the machine is information lossless. For this we need to construct a testing table for information lossless. If the machine is information lossless then and only a single input string can be found for a single beginning state and single final state.

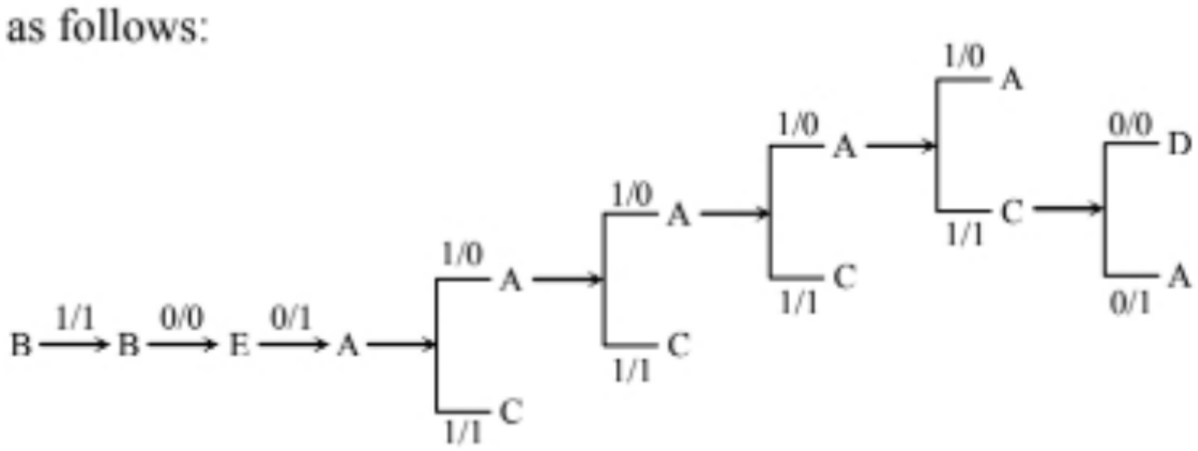
The testing table for information lossless:

PS	NS	
	$z=0$	$z=1$
A	–	(AC)
B	E	B
C	(AD	–
D	(BC)	–
E	A	B
AC	–	–
AD	–	–
BC	(AE)(DE)	–
AE	–	(AB)(BC)
DE	(AB)(AC)	–
AB	–	(AB)(BC)

The testing table does not contain any repeated entry. The machine is information lossless machine. The output successor table for the given machine:

PS	NS, i/p	
	$z=0$	$z=1$
A	–	(A, 0). C, 1)
B	E, 0	B, 1
C	(D, 0), (A, 1)	–
D	(C, 0), (B, 1)	–
E	A, 1	B, 0

Transition is as follows:



Beginning state B and final state D is obtained from one path with input string 10100010.

41. Define DFA. Derive the DFA for the regular language $0(0+1)^*1$ for the symbols $\Sigma = \{0, 1\}$. **[WBUT 2016]**

Answer:

1st part:

A deterministic finite automaton (DFA) is a 5-tuple. $(Q,\Sigma, \delta, q_0,F)$, where. Q is a finite set called the states, Σ is a finite set called the alphabet, $\delta : Q \times \Sigma \rightarrow Q$ is the transition function, $q0 \in Q$ is the start state, and $F \subseteq Q$ is the set of accept states.

2nd part:

42. Construct a minimum state automaton from the transition table given below:

PS	$x=0$	$x=1$
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
q_3	q_3	q_3
q_4	q_4	q_4
q_5	q_5	q_4

q_3, q_4, q_5 are final states. **[WBUT 2016]**

Answer:

In the finite automata, the states are $\{q_0, q_1, q_2, q_3, q_4, q_5\}$. Name this set as S_0.

$$S_0 : \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

All of the states are 0 equivalents.

In the finite automata, there are two types of states: final state and non-final states. So, divide the set of states into two parts, Q_1 and Q_2.

$Q_1 = \{q_0, q_1, q_2\}$ $Q_2 = \{q_3, q_4, q_5\}$

$S_1 : \{\{q_0, q_1, q_2\}, \{q_3, q_4, q_5\}\}$

The states belonging to the same subset are 1-equivalent because they are in the same set for string length 1. The states belonging to different subsets are 1-distinguishable.

For input 0 and 1, q_0 goes to q_1 and q_2, respectively. Both of the states belong to the same subset. For q_1 and q_2 with input 0 and 1, the next states are q_2, q_3, and q_2, q_4 respectively. For both of the states, for input 0, the next state belongs to one subset, and for input 1, the next state belongs to another subset. So, q_0 can be distinguished from q_1 and q_2.

The next states with input 0 and 1 for states q_3, q_4, and q_5, belong to the same subset. So, they cannot be divided

$S_2 : \{\{q_0\}, \{q_1, q_2\}, \{q_3, q_4, q_5\}\}$

q_0 is the single state in the subset. So, it cannot be divided.

For states q_1 and q_2 with input 0 and 1, for both of the cases, one state belongs to one subset and another state belongs to another subset. So, they cannot be divided.

The next states with input 0 and 1 for states q_3, q_4 and q_5 belong to the same subset. So, they cannot be divided. So in the next step,

$S_3 : \{\{q_0\}, \{q_1, q_2\}, \{q_3, q_4, q_5\}\}$

S_2 and S_3 are equivalent.

As step $(n-1)$ and step n are the same, there is no need of further advancement.

In the minimized automata, the number of states is 3.

The minimized finite automata is presented in tabular format as follows:

	Next State	
State	I/P=0	I/P=1
$\{q_0\}$	$\{q_1\}$	$\{q_2\}$
$\{q_1, q_2\}$	$\{q_2\}$	$\{q_3, q_4\}$
$\{q_3, q_4, q_5\}$	$\{q_3, q_4, q_5\}$	$\{q_3, q_4, q_5\}$

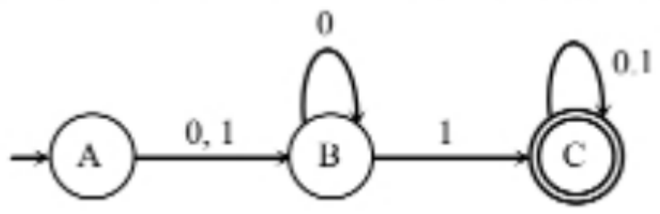
But $\{q_1\}, \{q_2\}$, and $\{q_3, q_4\}$ do not exist under the column of present state. They are not states of the minimized finite automata, but they are subset of the states. In the next state columns, by replacing the subsets by proper state, the modified table becomes

	Next State	
State	I/P=0	I/P=1
$\{q_0\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_3, q_4, q_5\}$
$\{q_3, q_4, q_5\}$	$\{q_3, q_4, q_5\}$	$\{q_3, q_4, q_5\}$

As q_0 is the beginning state of the original finite automata, $\{q_0\}$ will be the beginning state of minimized finite automata. As q_3, q_4, and q_5 are the final states of the original finite automata, the set of the states containing any of the states as element is final state. Here, all the states are contained in a single set $\{q_3, q_4, q_5\}$ and, therefore, it is the final state. By replacing $\{q_0\}$ as A, $\{q_1, q_2\}$ as B, and $\{q_3, q_4, q_5\}$ as C, the modified minimized finite automata becomes,

State	Next State I/P=0	I/P=1
$\rightarrow A$	B	B
B	B	C
(C)	C	C

The transitional diagram of the minimized finite automata is given in following figure.

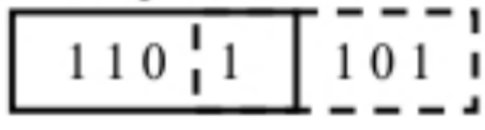


43. Design a two-input two-output sequence detector which generates an output 1 every time the sequence 1101 is detected. And for all other cases output 0 is generated. Overlapping sequences are also counted. [WBUT 2017, 2019]

Answer:

The sequence is 1101. We have to start from S_1. If we get input 0, then there is no chance, so it is confined in S_1 producing output 0. If we get input 1 then there is a chance to get 1101, so the control moves to S_2 producing output 0 (as we have not till got 1101 as input). In S_2 if we get 0 then there is a chance to get 1101 considering the last 1, so the control will be confined in S_2 producing output 0. In S_2 if we get 1 then there is a chance to get 1101 so the control moves to S_3 producing output 0.

In S_3 if we get 0, there is a chance to achieve the string 1101, so the control moves to S_4 producing the output 0. But in S_3 if we get 1 then there is no chance to get the string 1101, so in this case we have to start again from the beginning i.e., S_1. In S_4 also, if we get 0 then there is no chance of achieving the input 1101 so the control moves to the beginning producing output 0, but if we get 1 then the string 1101 is achieved, thus producing the output 1. As overlapping sequence is also accepted, the control moves to S_2, so that by getting 101, the sequence detector can produce 1.



The state diagram is given below –

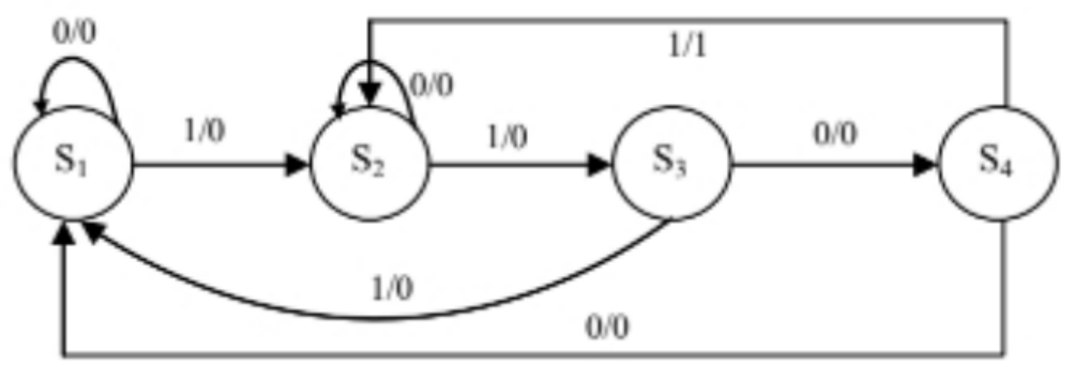


Figure 1

The state table for the sequence detector,

PS	NS	O/P
	X 0	1
S_1	S_1, 0	S_2, 0
S_2	S_3, 0	S_2, 0
S_3	S_4, 0	S_1, 0
S_4	S_1, 0	S_2, 1

44. Define Deterministic Finite Automata. What do you mean by NFA with -moves.
[WBUT 2018]

Answer:

Definition of DFA:

DFA is a 5-tuple.

$$(Q, \Sigma, \delta, q_0, F)$$

$Q \rightarrow$ finite set of states

$\Sigma \rightarrow$ finite set of alphabets

$\delta \rightarrow$ transition function

$$\delta : \Sigma \times Q \rightarrow Q$$

$q_0 \rightarrow$ start stable

$F \rightarrow$ sets of final states

NFA with λ-moves:

Let Q be finite set.

Let Σ be a finite set of symbols.

$\therefore$ δ be transition function defined as,

$$Q \times \Sigma U \{\lambda\} \rightarrow 2^Q$$

$q_0 \rightarrow$ be a state in Q

and A be a subset of Q.

$\therefore$ A is set of accepting states.

So, NFA with λ-moves, is a 5-tuple

$$\{Q, \Sigma, q_0, \delta, A\}$$

$\therefore$ Basically, any NFA is also a NFA with λ-move

45. Construct the language for the grammar $G=(\{S\},(a,b),S,P)$, with P given by

$$S \to aSb$$
$$S \to \lambda$$

[WBUT 2018]

Answer:

$G=(\{S\},\{a,b\},S,P)$ with P given by

$$S \to aSb,\ S \to \lambda$$

$\therefore$ $S \to a\,\underline{S}\,b \to a > b \to ab$

$\therefore$ $S \to a\,\underline{S}\,b \to aa\,\underline{S}\,bb \to aa > bb \to aabb$

$\therefore$ $S \to a\,\underline{S}\,b \to aa\,\underline{S}\,bb \to aaa\,\underline{S}\,bbb \to aaabbb$

So, the Language is → "Equal no. of a 's followed by equal no. of b 's with Null".

$\therefore$ $L(G)=\{a^n b^n;\ n \geq 0\}$

46. What is Homomorphism? Find the homomorphic image of $L=\{aa, aba\}$ where h is defined as $h(a)=ab$ and $h(b)=bbc$. **[WBUT 2018]**

Answer:

1st Part:

Homomorphism:

A homomorphism is a map between two algebraic structures of the same type, that preserves the operations of the structures.

$\therefore$ $f: A \to B$ where two sets A, B having same structure.

Then, say '*' is a binary operation.

$\Rightarrow$ $f(x*y)=f(x)*f(y)$

for every pair of $(x,y) \in A$

2nd Part:

$\therefore$ $L=\{aa,\ aba\}$

where $h(a)=ab,\ h(b)=bbc$

$\therefore$ $h(a\ \ a)=h(a)\ \ h(a)=ab\ \ ab$

$h(a\ \ b\ \ a)=h(a)\ \ h(b)\ \ h(a)=ab\ \ bbc\ \ ab$

$\therefore$ Homomorphic image of L is —

$\{ab\,ab,\ ab\ \ bbc\ \ ab\}$

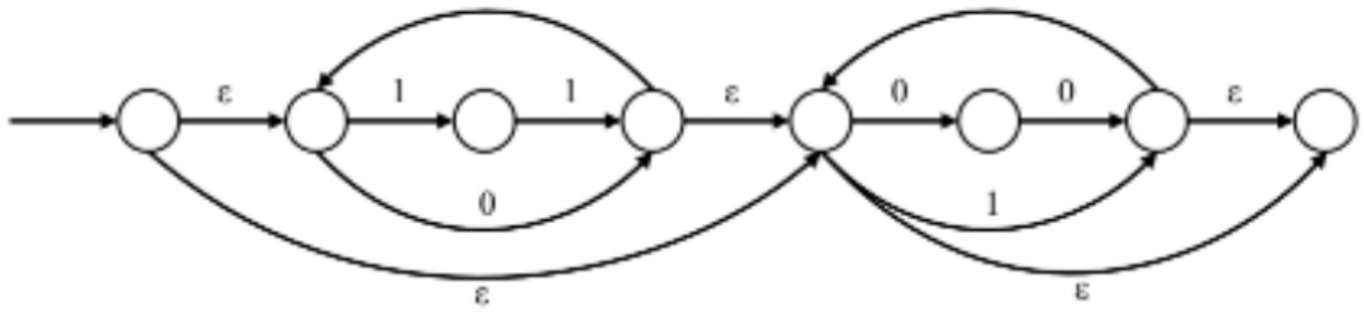
47. Construct a DFA for the following regular expression: **[WBUT 2019]**

$$R = (11+0)^*(00+1)^*.$$

Answer:

Construction of DFA:

$$R = (11+0)^*(00+1)^*$$



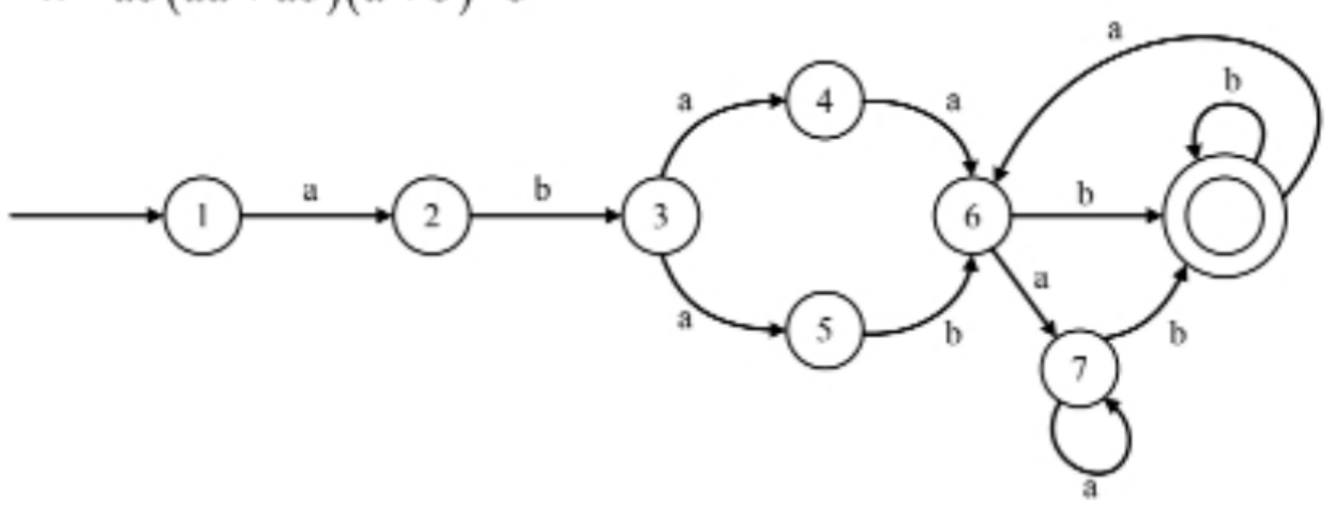
48. Construct a Finite Automata Equivalent to the Regular expression:

[WBUT 2019]

$$L = ab(aa+ab)(a+b)^*b$$

Answer:

$$L = ab(aa+ab)(a+b)^*b$$



Long Answer Type Questions

1. a) In response to an unknown input sequence, the machine of table-4 produces the output sequence 1 1 1 0 0 0 0 0 1 0.
Find the input sequence to the machine if it is known that its initial state is A and final state is F.

PS	NS, Z	
	x = 0	x = 1
A	B, 1	C, 0
B	D, 1	B, 1
D	E, 1	B, 0
F	A, 0	E, 0
F	F, 0	D, 1
F	D, 0	A, 1

Table-4

b) Find the minimal inverse machine of the machine given in table-5. [WBUT 2004]

PS	NS, Z	
	x = 0	x = 1
A	C, 0	D, 1
B	D, 0	C, 1
C	A, 0	B, 0
D	C, 1	D, 1

Table-5

OR,

Show that the following FSM is information lossless of finite order:

PS	NS, Z	
	x = 0	x = 1
A	C, 0	D, 1
B	D, 0	C, 1
C	A, 0	B, 0
D	C, 1	D, 1

Table-5

Also find its order of information losslessness. [WBUT 2008]

Answer:

a) We use the backward sequencing to get

A B B D E F D A C E F
1 1 1 0 0 0 0 0 1

Hence, the input sequence is

0 1 0 1 0 0 0 1 0 0

b)

D	C,1	D,1

Machine-M

This machine is loss less of order 3. If we know the initial state and the values of three successive outputs, which are produced due to transition from this initial state, then we can determine the first input to the machine.

Now the present states of the inverse machine can be defined by a set of triples, which is denoted by $(S(t), Z(t+1), Z(t+2))$ where $S(t)$ is the possible states of the original machine M and $Z(t+1)$, $Z(t+2)$ are 2 outputs.

For the above machine we have the following triples:

(A,0,0) (B,0,1) (C,0,0) (D,1,0)
(A,1,1) (B,1,0) (C,0,1) (D,1,1)

The rest of the triples are not defined.

Thus the inverse machine has present states defined by the eight triples shown above.

Now the next state of the inverse machine can also be defined by triples.

The first member of this triple is the state to which the machine M will go consuming the first input symbol.

The second member is nothing but the third member of the present state of the inverse machine and finally the third member is the present output of M.
The state table of the corresponding inverse machine is shown below:
Machine-M^i

PS	NS, X	
	z = 0	z = 1
(A,0,0)	(C,0,0),0	(C,0,1),0
(A,1,1)	(D,1,0),1	(D,1,1),1
(B,0,1)	(D,1,0),0	(D,1,1),0
(B,1,0)	(C,0,0),1	(C,0,1),1
(C,0,0)	(A,0,0),0	(B,0,1),1
(C,0,1)	(B,1,0),1	(A,1,1),0
(D,1,0)	(C,0,0),0	(C,0,1),0
(D,1,1)	(D,1,0),1	(D,1,1),1

2. a) Construct a minimum state automaton equivalent to a given automation *M* whose transition table is given below: **[WBUT 2007]**

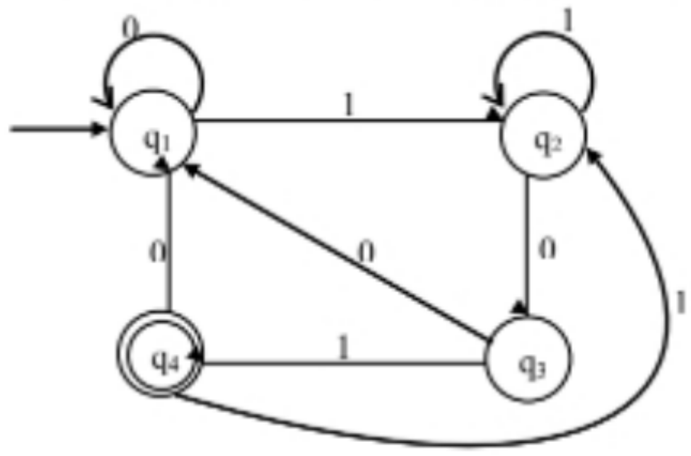
State	*Input*	
	a	*b*
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
(q_6)	q_1	q_3

Answer:

q_1	×					
q_2	×	×				
q_3	×	×	×			
q_4	×	×	×	×		
q_5	×	×	×	×	×	
q_6	×	×	×	×	×	×
	q_0	q_1	q_2	q_3	q_4	q_5

Since all states are distinguishable, the DFA is already minimized.

b) Find the regular expression corresponding to the following figure: [WBUT 2007]



Answer:

$0^*1^+(0^+1^+)^*0\,1$

3. a) Find the equivalence partition for the machine shown in Table. Show a standard form of the corresponding reduced machine and find a minimum-length sequence that distinguishes state *A* from state *B*. [WBUT 2007, 2012, 2015]

PS	*NS, Z*	
	X = 0	*X* = 1
A	B, 1	H, 1
B	F, 1	D, 1
C	D, 0	E, 1
D	C, 0	F, 1
E	D, 1	C, 1
F	C, 1	C, 1
G	C, 1	D, 1
H	C, 0	A, 1

Answer:

P_0 = (A B C D E F G H)
P_1 = (A B E F G) (C D H)
P_2 = (A B) (E F G) (C D H)
P_3 = (A) (B) (E F G) (C D H)
P_4 = (A) (B) (E F G) (C D) (H)
P_5 = (A) (B) (E F G) (C D) (H) ← Final equivalent partition

Let α = (A,) β = (B), γ = (E F G),
δ = (C D), $\in$ = (H).
Hence the standard form machine is

PS	**NS, Z**	
	x = 0	**x = 1**
α	β, 1	∈, 1
β	γ, 1	δ, 1
γ	δ, 0	δ, 1
δ	δ, 0	γ, 1
∈	δ, 1	α, 1

There are no 1-length or 2-length sequence that distinguishes A and B.

The sequence 000 distinguishes A and B

In state A, the output is 111

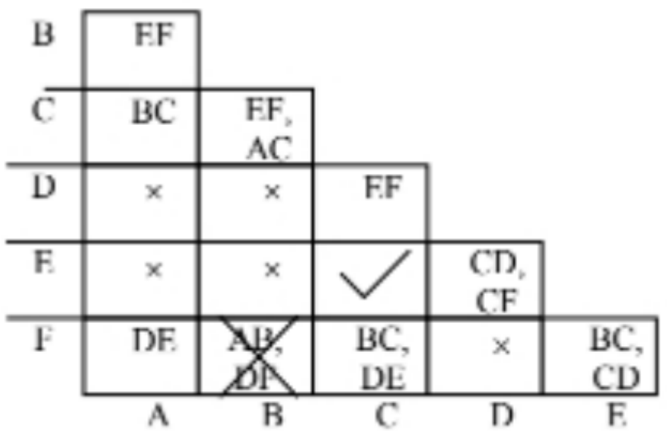
In state B, the output is 110

b) For the incompletely specified machines shown in Table-2 find a minimum-state reduced machine containing the original one. [WBUT 2007]

Table - 2

PS	*NS, Z*	
	I_1	I_2
A	E, 0	B, 0
B	F, 0	A, 0
C	E, –	C, 0
D	F, 1	D, 0
E	C, 1	C, 0
F	D, –	B, 0

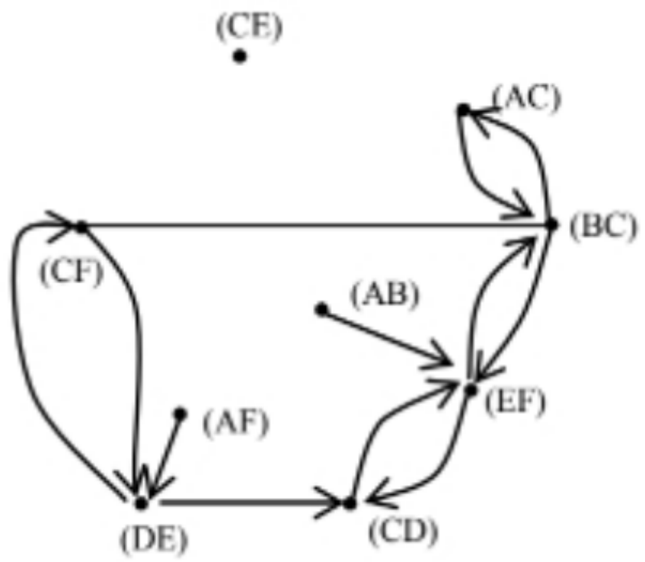
Answer:



Procedure

- Write down all compatible pair in respective cell.
- An '×' is inserted in cell where the states have conflicting outputs. Here A and D have conflicting output. Similarly AE, BD, BE having conflicting output.
- Insert a check mark in cell CE because state E contain State C.
- Cross those cells having incompatible pair. Here cell BF contain incompatible pair DF so ignored the cell BF.

Compatibility Graph



The set {(AB), (AC), (BC), (CD), (EF)} → {(ABC), (CD), (EF)} is a minimal cover of the above machine.

4. a) For the following machine shown in Table determine whether or not it is lossless. If it is lossy, find the shortest output sequence produced by two different input sequences with the same initial and final states. If it is lossless, determine its order.

PS	*NS, Z*	
	X = 0	*X* = 1
A	A, 0	B, 0
B	C, 0	D, 0
C	D, 1	C, 1
D	B, 1	A, 1

b) Design a minimal inverse of the machine shown in Table **[WBUT 2007]**

PS	*NS, Z*	
	X = 0	*X* = 1
A	C, 0	D, 1
B	D, 0	C, 1
C	A, 0	B, 0
D	C, 1	D, 1

Answer:

a) As another illustration, the above test is applied to machine M_9 of Table (i). This machine is shown to be lossless of order 3, since its testing graph (Fig. a) is loop-free and the longest path is of length 1.

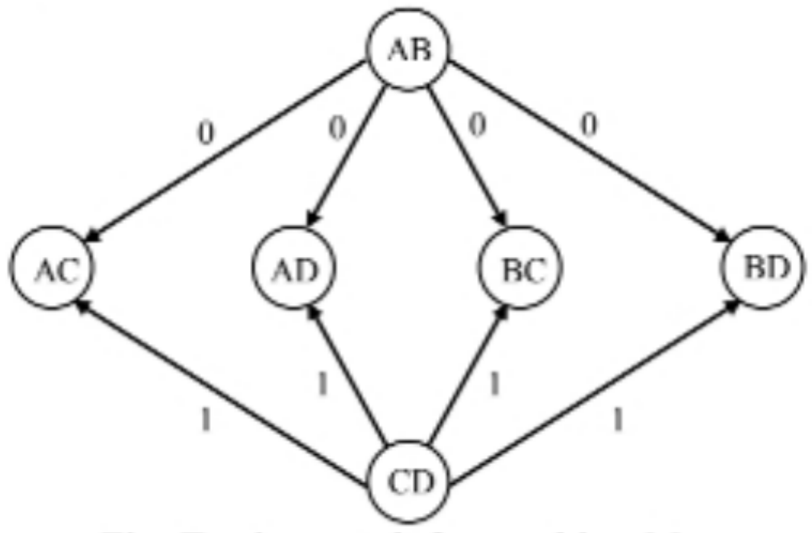


Fig: Testing graph for machine M_9

Table (i): Machine M_9

PS	*NS, z*	
	$x = 0$	$x = 1$
A	*A*, 0	*B*, 0
B	*C*, 0	*D*, 0
C	*D*, 1	*C*, 1
D	*B*, 1	*A*, 1

Table (ii): Testing table for M_9

PS	$z = 0$	$z = 0$
A	*(AB)*	*B*, 0
B	*(CD)*	*D*, 0
C	–	*C*, 1
D	–	*A*, 1
AB	*(AC)(AD) (BC)(BD)*	–
CD	–	*(AC)(AD) (BC)(BD)*

b) We have the following triples:

(A,0,0)	(B,0,1)	(C,0,0)	(D,1,0)
(A,1,1)	(B,1,0)	(C,0,1)	(D,1,1)

The rest of the triples are not defined.
Thus the inverse machine has present states defined by the eight triples shown above.
Now the next state of the inverse machine can also be defined by triples.
The first member of this triple is the state to which the machine M will go consuming the first input symbol.
The second member is nothing but the third member of the present state of the inverse machine and finally the third member is the present output of M.
The state table of the corresponding inverse machine is shown below: -
Machine-M^i

PS	NS, X	
	z = 0	z = 1
(A,0,0)	(C,0,0),0	(C,0,1),0
(A,1,1)	(D,1,0),1	(D,1,1),1
(B,0,1)	(D,1,0),0	(D,1,1),0
(B,1,0)	(C,0,0),1	(C,0,1),1
(C,0,0)	(A,0,0),0	(B,0,1),1
(C,0,1)	(B,1,0),1	(A,1,1),0
(D,1,0)	(C,0,0),0	(C,0,1),0
(D,1,1)	(D,1,0),1	(D,1,1),1

5. a) Design a 2-input 2-output Mealy machine, which takes as input a binary stream and generates an output of 1 only when a sequence of the pattern 01011 is found in the input stream. Design should be clearly justified. [WBUT 2007, 2010]

Answer:

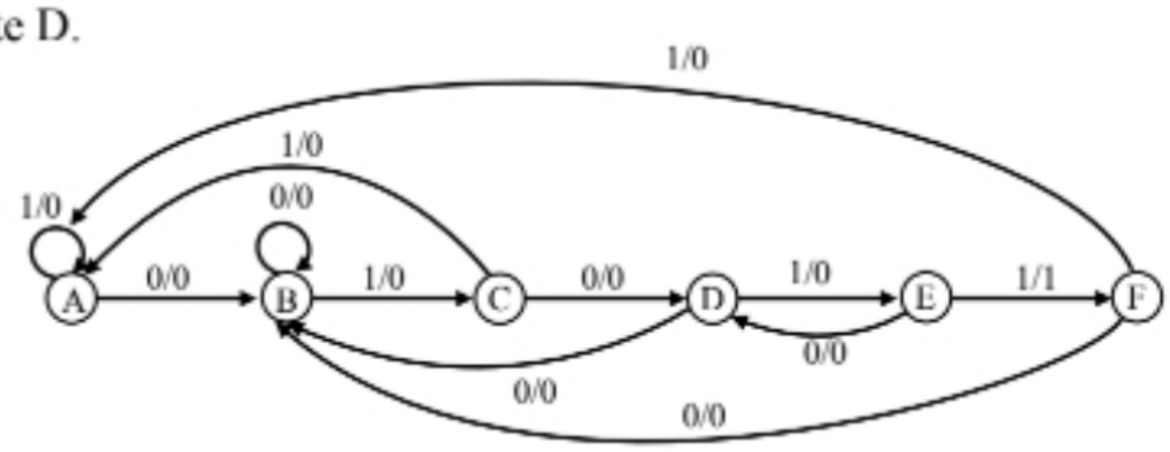
The required Mealy machine is given below:

PS	*NS, Z*	
	X = 0	*X* = 1
A	B, 0	A, 0
B	B, 0	C, 0
C	D, 0	A, 0
D	B, 0	E, 0
E	D, 0	F, 1
F	B, 0	A, 0

Justification of Design

The forward moving (i.e., A → B → C → D → E → F) transitions can be trivially justified. Also justified are 1 transitions from states A, C and F to state A because this indicates the sequence has been broken completely and scanning must begin again from state A.
We note that the sequence starts with a 0 and upon receiving the first 0, the state is B. Hence, if the sequence breaks in any state B, D or F upon receiving a 0, we should count this 0 as the first possible symbol of the input and begin from state B.

Getting a 0 in the state E is a special case. State E is reached upon receiving the subsequence 0101. If a 0 is received, the sequence is 01010, the last three symbols of which (i.e. 010) is the first three symbols of the sequence to be recognized. But the state which is reached upon receiving subsequence 010 is D. Hence the 0 transition from state E should be state D.



State	Basic String
A	C
B	0
C	01
D	010
E	0101
F	01011

Pending	Basic String Possible	State recognizing largest suffix of basic string possible
A,1	1	A
B,0	00	B
C,1	011	A
D,0	0100	B
E,0	01010	D
F,0	010110	B
F,1	010111	A

b) From the Mealy machine above find the equivalent Moore machine. [WBUT 2007]

Answer:

The Mealy machine designed in part a) produces output 1 only when there is a transition into state F.

Hence the equivalent Moore machine is:

PS	*NS*		*Output*
	X = 0	*X* = 1	
A	B	A	0
B	B	C	0
C	D	A	0
D	B	E	0
E	D	F	0
F	B	A	1

c) Check whether the Mealy machine you obtained is a minimal one or not. Give proper justification to your answer. [WBUT 2007]

Answer:

The designed Mealy machine is NOT minimal. If we begin with the initial partition P0 = (ABCDEF), we will get P1 = (ABCDF)(E) because only in state E we get output 1.

Thereafter, states D, C and B form independent sets but states A and F are indistinguishable.

The minimal machine is therefore:

PS	***NS, Z***	
	***X* = 0**	***X* = 1**
A	B, 0	A, 0
B	B, 0	C, 0
C	D, 0	A, 0
D	B, 0	E, 0
E	D, 0	A, 1

6. a) Find a reduced grammar equivalent to the grammar **[WBUT 2009]**

$S \to aAa, A \to bBB, B \to ab, C \to aB$.

Answer:

Clearly C is a useless symbol. Eliminating C, we get,

$$S \to aAa,\ \ A \to bBB,\ B \to ab$$

Now we replace all occurrences of B on rhs of production using $B \to ab$, getting:

$$S \to aAa,\ \ A \to babab$$

Now we similarly substitute A, getting

$$S \to a\ ba\ ba\ ba$$

which is the equivalent reduced grammar.

b) Explain the concept of 2-way finite automata. **[WBUT 2009]**

Answer:

In a one-way (deterministic) finite automata, after every state transition, the input "head" moves to the 'next' input, i.e., to the 'right'. In a two way finite automata, the "head' can move either to the "right" or to the "left".

More formally:

A 2DFA is a quintuple $M = (Q, \sum, \delta, q_0, F)$ where $\sum$ is a finite set of input symbols, Q is a finite set of states, $q_0 \in Q$ is a "start state" and $F \subseteq Q$ is a set of accepting states. δ is a map from $Q \times \sum$ to $Q \times \{L, R\}$. If $\delta(q,a) = (p,L)$, then in state q, scanning symbol a, the 2DFA enters state p and moves its head "left", if $\delta(q,a) = (p,R)$, the $2DFA$ enters state p and moves "right".

A 2DFA accepts a string if it moves "right" beyond the last symbol, at the same time entering an accepting state.

It can be proved that if L is a language accepted by a 2DFA then L is a regular set.

7. a) What do you mean by *k*-equivalent states? [WBUT 2009, 2011]
b) Minimize the following machine by partitioning the distinguishable states:

Present state	$i/p=0$		$i/p=1$	
	Next State	o/p	Next State	o/p
A	E	0	D	1
B	F	0	D	0
C	E	0	B	1
D	F	0	B	0
E	G	0	F	1
F	B	0	C	0
G	C	1	H	0
H	A	1	G	0

c) Give definition of lossy and lossless machine. [WBUT 2009]

Answer:

a) Two states S_i and S_j of a finite state machine are said to be K equivalent it there exists No input sequence of length K or less that produces different outputs depending upon whether S_i or S_j is the initial state.

b) Initial partition $P_0 = (ABCDEFGH)$

Partition of 1-distinguishable states = $P_1 = (ACE)(BDF)(GH)$

Partition of 2-distinguishable states = $P_2 = (AC)(E)(BD)(F)(GH)$

Partition of 3-distinguisbable states = P_3 = None

So, minimal machine has 5 states.

Let us map: $(AC) \to P \quad (E) \to Q \quad (BD) \to R \quad (F) \to S$ and $(GH) \to T$

The minimal machine is:

PS	NS , z	
	$x=0$	$x=1$
P	Q, 0	R, 1
Q	T, 0	S, 1
R	S, 0	R, 0
S	R, 0	P, 0
T	P , 1	T, 0

c) A machine M is said to be (information) lossless if the knowledge of the initial state, the output sequence and the final state is sufficient to determine unequally the input sequence.

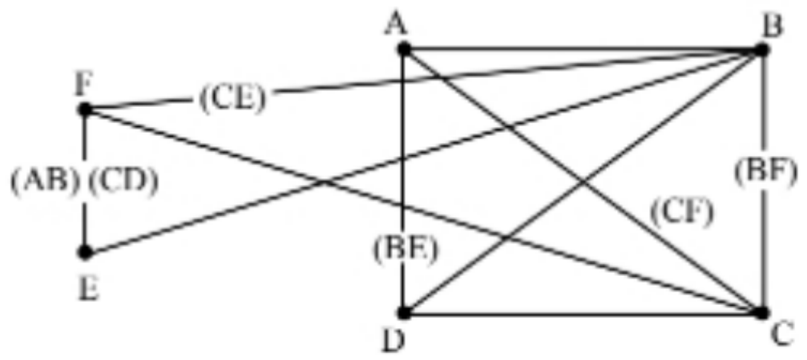
A machine that is not lossless is said to be lossy.

8. Draw the merger graph, merger table, compatibility graph and then minimize the following machine: [WBUT 2009]

Present state	Next State, o/p		Next State, o/p	
	$i/p=0$	$i/p=1$	$i/p=2$	$i/p=3$
A	–	C, 1	E, 1	B, 1
B	E, 0	–	–	–
C	F, 0	F, 1	–	–, 1
D	–	–	B, 1	–
E	–	F, 0	A, 0	D, –
F	C, –	–	B, 0	C, 1

Answer:

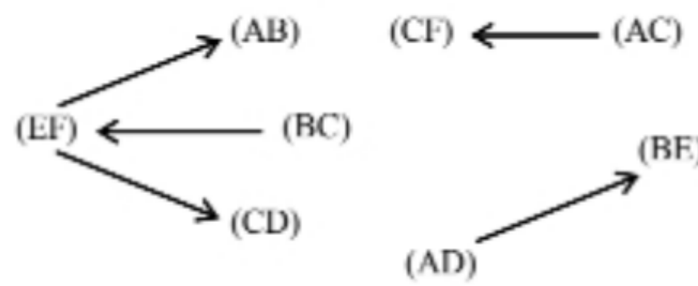
Merger Graph



Merger Tale

B	✓			
C	CF	EF		
D	BE	✓	✓	
E	×	✓	×	×
F	×	CE	✓	×	AB CD
	A	B	C	D	E

Compatibility Graph



From the compatibility graph, we get {(AB), (CD), (EF)} as a closed covering of compatibles.

Minimal Machine

PS	NS, z			
	i/p = 0	i/p = 1	i/p = 2	i/p = 3
$(AB)\rightarrow\alpha$	γ,0	β,1	γ,1	α,1
$(CD)\rightarrow\beta$	γ,0	γ,1	α,1	–,1
$(EF)\rightarrow\gamma$	β,–	γ,0	α,0	β,1

9. a) What is regular language? [WBUT 2009, 2010, 2018]

Answer:
A regular language is a formal language defined over a finite alphabet such that ever string in the language can be:

a) Accepted by some NFA/DFA
b) Described by a regular expression
c) Generated by a Regular (i.e. Type-3) Grammar.

b) Find regular expressions over $\sum = \{a, b\}$ for the languages defined as follows:

i) $L1 = \{a^m b^m : m > 0\}$

ii) $L2 = \{a^{2n} b^{2m+1} \mid n \geq 0, m\, n \geq 0\}$

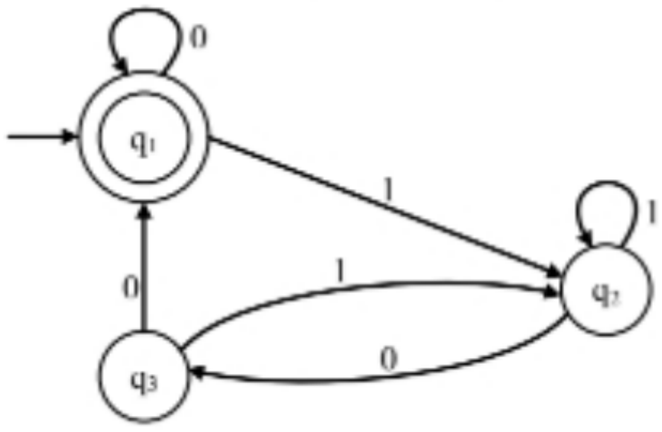
iii) $L3 = \{b^m a b^n : m > 0, n > 0\}$ **[WBUT 2009, 2010, 2018]**

Answer:
i) Not possible since the language is not regular.
ii) $(aa)*b(bb)*$
ii) $bb*abb*$

c) Find the regular expression for following transition graph. [WBUT 2009, 2010]

OR,

Construct the regular expression corresponding to the state diagram given below: [WBUT 2013]



Answer:
The state equations are

$q_1 = \varepsilon + q_1 0 + q_3 0$ ....(1)

$q_2 = q_1 1 + q_2 1 + q_3 1$ ....(2)

$q_3 = q_2 0$ ....(3)

Substituting (3) in (2) we get:

$q_2 = q_1 1 + q_2 (1 + 01)$ ....(4)

We know that if $R = Q + RP$, then $R = QP^*$

Hence, from (4),

$q_2 = q_1 1(1 + 01)^*$ ....(5)

Substituting (3) and (5) in (1)
We get:

$$q_1 = \varepsilon + q_1 0 + q_2 00$$

$$= \varepsilon + q_1\left(0 + 1(1+01)^*00\right)$$

$$\text{or } q_1 = \varepsilon\left(0 + 1(1+01)^*00\right)^*$$

$$= \left(0 + 1(1+01)^*00\right)^*$$

Which is the RE for the given transition diagram.

10. a) What is information lossless machine? **[WBUT 2010]**
b) Consider the machine shown in the following table: **[WBUT 2010, 2014]**

Present State	Next State	
	X = 0	X = 1
A	A, 1	C,1
B	E, 0	B, 1
C	D, 0	A, 0
D	C, 0	B, 0
E	B, 1	A, 0

Is this machine information lossless of finite order? If yes, find the order μ.

Answer:

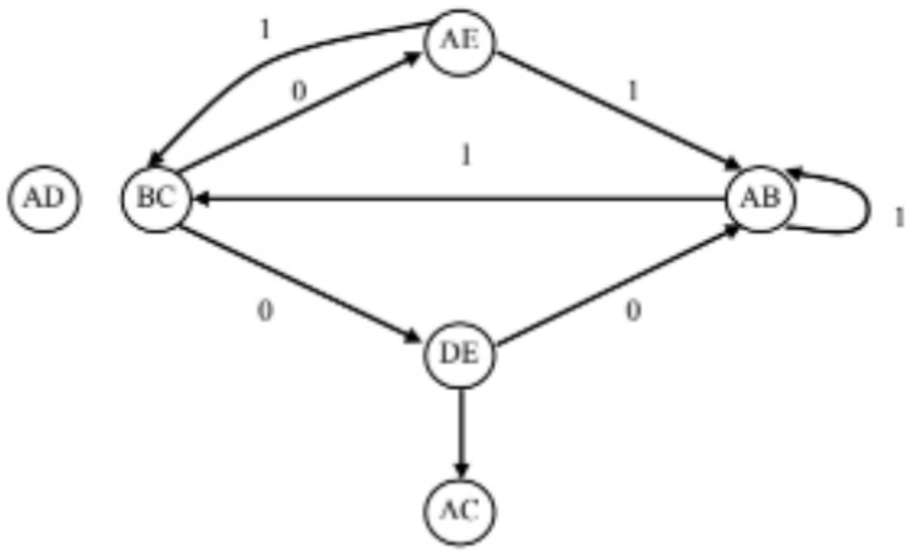
a) A machine is to be *(information) lossless* if the knowledge of the initial state, the output sequence, and the final state is sufficient to determine uniquely the input sequence.

b) ***Step 1:*** Check each row of the state table for an appearance of two identical next-state entries associated with the same output symbol.
Here (AC*) is* a compatible pair, since both *A* and *C* are the output 1-successors of *A*. Similarly, the pairs *(AD)* and *(BC)* are compatible pairs.

Step 2: If no identical entries appear, the next step is to construct the output successor table.
These pairs are used as row headings for the lower part of the testing table. Pairs *(AE)* and *(DE),* which are implied by *(BC),* are now made row headings, and so on. Note that contrary to the testing procedure for finite output memory, the testing table for information losslessness does not necessarily include all distinct pairs of states, but only the compatible ones.

Testing table of machine M

PS	z = 0	z = 1
A	-	(AC)
B	E	B
C	(AD)	-
D	(BC)	-
E	A	B
AC	-	-
AD	-	-
BC	(AE)(DE)	-
AE	-	(AB)(BC)
DE	(AB)(AC)	-
AB	-	(AB)(BC)



Testing graph for machine *M*

The testing graph *G* of machine *M is* derived from the lower half of the testing table. The testing table does not contain any compatible pair consisting of repeated entries. So the machine is **information lossless.**

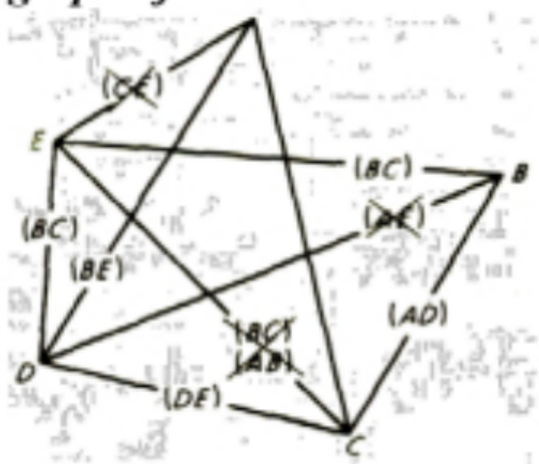
11. Consider the following machine: **[WBUT 2010]**

Present State	Next State			
	I_1	I_2	I_3	I_4
A	-	-	E, 1	-
B	C, 0	A, 1	B, 0	-
C	C, 0	D, 1	-	A, 0
D	-	E, 1	B, -	-
E	B, 0	-	C, -	B, 0

i) Draw the merger graph.
ii) Draw the merger table.
iii) Draw the compatibility graph.
iv) Find the minimal closed covering with justification.

Answer:

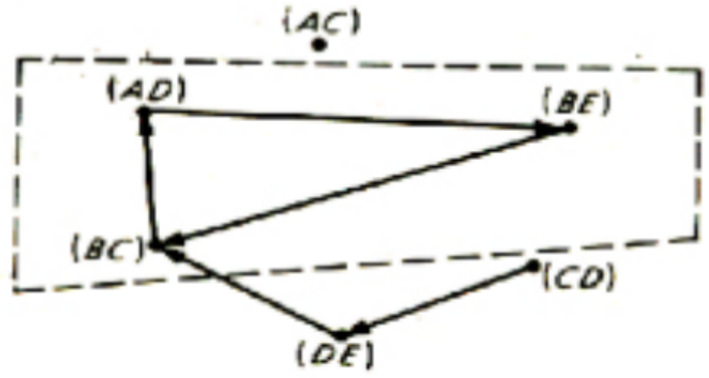
Step 1: First draw the merger graph of machine.



The set of maximal compatibles derives from the merger graph contain four members and is given by {(ACD),(BC),(BE),(DE)}

Step 2: *The compatible pairs and their implied pairs are usually obtained by the merger graph, and since a set of states is a compatible if and only if every pair of states in that set is compatible, then for a given , the set of compatible* **pairs defines uniquely the entire set of compatibles .**

Here from the above merger graph, an arc leads from vertex *(AD)* to vertex *(BE)* because the compatibility of *(BE)* is implied by that of *(AD)*. No arcs emanate from *(AC)* since no other compatible is implied by it, and so on.



Step 3: Select closed subgraph in the compatible graph.

12. Construct the minimum state automata equivalent to given automata M defined below: **[WBUT 2010]**

Σ / states	a	b
$\rightarrow q_0$	q_5	q_1
q_1	q_2	q_6
$*q_2$	q_2	q_0
q_4	q_5	q_7
q_5	q_6	q_2
q_6	q_4	q_6
q_7	q_2	q_6

(* q_2 indicates that q_2 is the final state)

Answer:

q_1	×					
q_2	×	×				
q_4		×	×			
q_5	×	×	×	×		
q_6	×	×	×	×	×	
q_7	×		×	×	×	×
	q_0	q_1	q_2	q_4	q_5	q_6

$$\{q_0, q_4\}, \{q_1, q_7\}, \{q_2\}, \{q_5\}, \{q_6\}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\rightarrow A \quad B \quad C^* \quad D \quad E$$

	a	b
→ A	D	B
B	C	E
C*	C	A
D	E	C
E	A	E

13. a) What do you mean by Distinguishable and Indistinguishable state?
[WBUT 2011, 2012, 2016]

Answer:

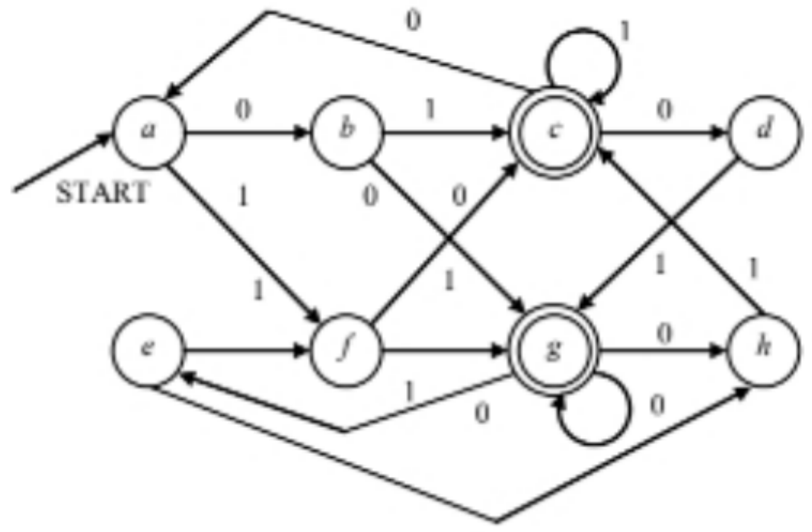
Two distinct states, Si, and S_j , of a machine *M* are *distinguishable* if and only if there exists at least one finite input sequence which, when applied to *M*, causes different output sequences, depending on whether S_i or S_j is the initial state.

The sequence which distinguishes these states is called a distinguishing sequence of the pair (S_i, S_j).

If no distinguishing sequence exists for the pair (S_i, S_j), then the states are said to be indistinguishable.

The principle of state minimization of a finite state machine is based on the principle of identifying equivalent classes for states which are indistinguishable in the original machine and then assigning one state per class in the reduced machine.

b) Use Myhill Nerode Theorem to minimize the following finite automata:
[WBUT 2011, 2012]



Answer:

g	×						
f		×					
e		×					
d		×					
c	×		×		×		
b		×				×	
a						×	
	h	g	f	e	d	c	b

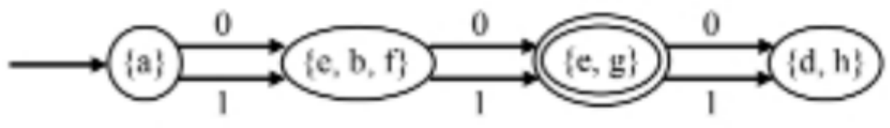
Next we can see that delta (a, 0) = b, and delta (d, 0) = g and (b, g) is marked, so, (a, b) gets marked too. Similarly we can mark,

g	×						
f		×					
e		×	×				
d		×		x			
c	×		×		x		
b	×	×		x		x	
a			x		x	x	x
	a	g	f	e	d	c	b

In the next round we can find that delta (d, 1) = g and delta (g, 1) = e where, (g, e) is marked. So, (d, f) will be marked. Similarly we can mark

g	×						
f		×					
e		×	×				
d	×	×	×	×			
c	×		×	×	×		
b	×	×		×		×	
a	×	×	×	×	×	×	×
	h	g	f	e	d	c	b

Here {a} is completely distinguishable state.
Hence the indistinguishable states are (c, g), (f, h), (e, h), (b, f), (b, d). This can be refined to get indistinguishable sets of states as: {e, b, f}, {e, g}, {d, h}
We get the minimized DFA as shown in figure,



14. a) Construct a DFA from the NFA given below:

State/Σ	I/P	
	0	1
→ Q_0	Q_0, Q_3	Q_0, Q_4
Q_3	Q_f	.....
Q_4	.....	Q_f
Q_f (Final State)	Q_f	Q_f

b) Construct λ NFA for the regular expression $(0+1)^* 1(0+1)$. [WBUT 2013]

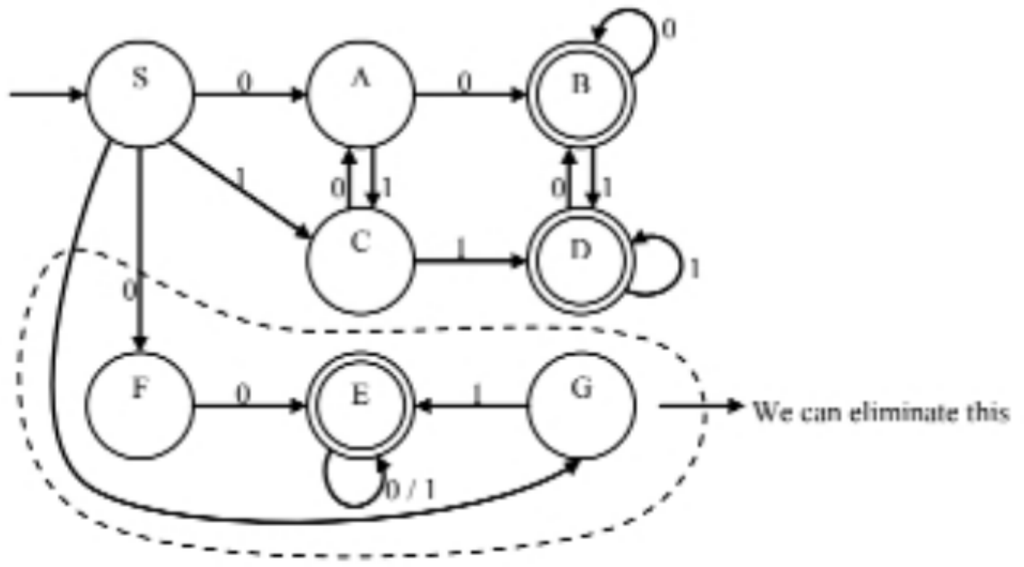
Answer:

a) DFA from the NFA: Here Null transitions are replaced by New (N) states

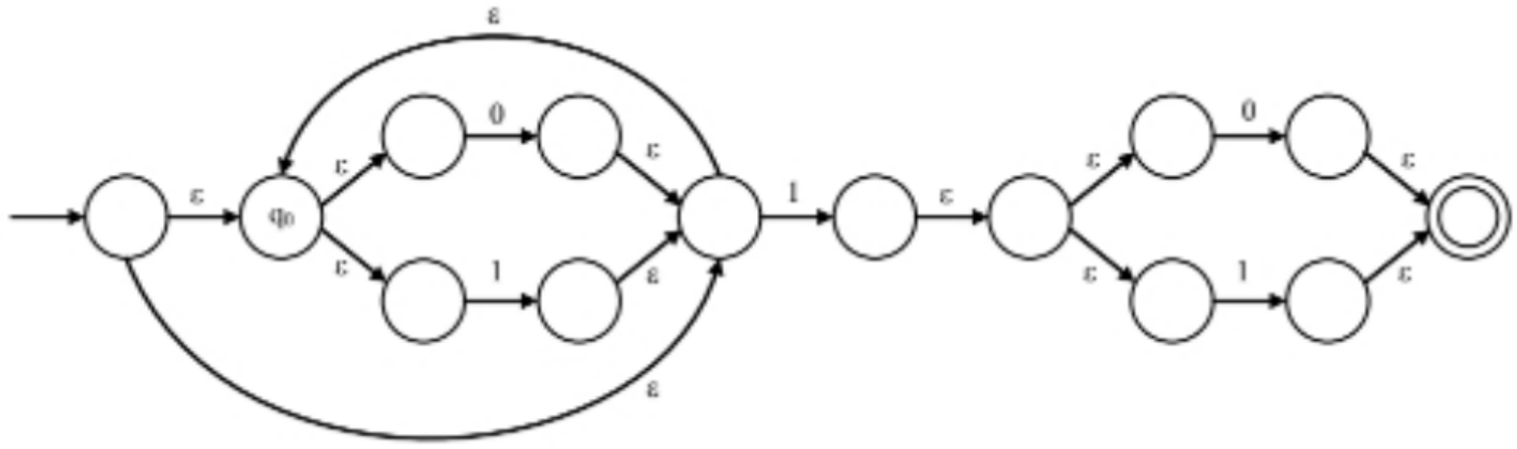
States	Input	
	0	1
Q_0	$\{Q_0, Q_3\}$	$\{Q_0, Q_4\}$
$\{Q_3\}$	Q_f	N
Q_4	N	Q_f
$\{Q_f\}$	Q_f	$\{Q_f\}$
$\{Q_0, Q_4\}$	$\{Q_0, Q_3\}$	$\{Q_0, Q_4, Q_f\}$
$\{Q_0, Q_3\}$	$\{Q_0, Q_3, Q_f\}$	$\{Q_0, Q_4\}$
$\{Q_0, Q_3, Q_f\}$	$\{Q_0, Q_3, Q_f\}$	$\{Q_0, Q_4, Q_f\}$
$\{Q_0, Q_4, Q_f\}$	$\{Q_0, Q_3, Q_f\}$	$\{Q_0, Q_4, Q_f\}$

The following are the states that have been renamed.

$[Q_0] = S$
$[Q_0, Q_3] = A$
$[Q_0, Q_3, Q_f] = B$
$[Q_0, Q_4] = C$
$[Q_0, Q_4, Q_f] = D$
$[Q_3] = F$
$[Q_4] = G$
$[Q_f] = E$



b)



15. Minimize the following machine by applying Myhill-Nerode theorem. [WBUT 2013]

PS	NS	
	X = a	X = b
→A	B	E
B	C	D
C	H	I
D ○	I	H
E ○	F	G
F	H	I
G ○	H	I
H ○	H	H
I	I	I

Answer:

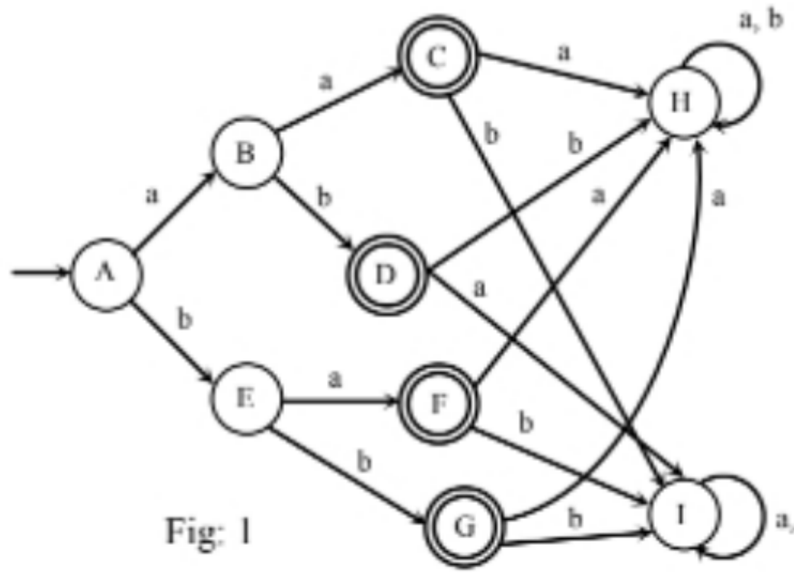


Fig: 1

There is no unreachable state in Fig. 1 so we can directly apply Myhill-Nerode theorem as follows:

$Q = \{A, B, C, D, E, F, G, H, I\}$

$\sum \{a, b\}$

Initial state $= A$

$F = \{C, D, F, G\}$

$\{Q - F\} = \{A, B, E, H, I\}$

We use an ordered $F \times F = \{(C,D),(C,F),(C,G),(D,F),(C,G),(F,G)\}$

and

$$(Q-F)\times(Q-F) = \{(A,B),(A,E),(A,H),(A,I),(B,E),(B,H),(B,I),$$
$$(E,H),(E,I),(H,I)\}$$

Transition table will be following:

$\delta / \sum$	a	b
$\rightarrow A$	B	E
B	C	D
$*C$	H	I
$*D$	I	H
E	F	G
$*F$	H	I
$*G$	H	I
H	H	H
I	I	I

Step 1: Now build the matrix labeling the "p" rows A,B,C … and labeling the "q" column A,B,… and put the dashes on the diagonal and below the diagonal

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
A	–								
B	–	–							
C	–	–	–						
D	–	–	–	–					
E	–	–	–	–	–				
F	–	–	–	–	–	–			
G	–	–	–	–	–	–	–		
H	–	–	–	–	–	–	–	–	
I	–	–	–	–	–	–	–	–	–

Fig: 2

Step 2: Now mark X at (p, q) in upper triangle such that p in F (Final states) and q in (Q – F) or (non-final states) as subscript of p is lower than the subscript of q or p in (Q – F) and q in F and p is lower than q.

$$F = \{C, D, F, G\}$$

$$Q - F = \{A, B, E, H, I\}$$

$$F \times (Q - F) \text{ or } (Q - F) \times F \text{ such that p is lower than q.}$$

$$= \{(C,E),(C,H),(C,I),(D,E),(D,H),(D,I),(F,H),(F,I),(G,I),(G,H),$$
$$(A,C),(A,D),(A,F),(A,G),(B,C),(B,D),(B,F),(B,G),(E,F),(E,G)\}$$

Now mark × in the matrix given in Fig: 3.

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
A	–		×	×		×	×		
B	–	–	×	×		×	×		
C	–	–	–		×			×	×
D	–	–	–	–	×			×	×
E	–	–	–	–	–	×	×		
F	–	–	–	–	–	–		×	×
G	–	–	–	–	–	–	–	×	×
H	–	–	–	–	–	–	–	–	
I	–	–	–	–	–	–	–	–	–

Fig: 3

Step 3: Mark 'x' and ' 0 ' in the matrix:

First we are considering Q – F

$Q-F=\{A,B,E,H,I\}$, so ordered pair of

$$(Q-F)\times(Q-F)=\{(A,B),(A,E),(A,H),(A,I),(B,E),(B,H),(B,I),(E,H),(E,I),(H,I)\}$$

* Now we will select (p, q) from (Q-F) x (Q-F) and find (r,s) as

$r=\delta(p,a),s=\delta(q,a)$, if (r, s) is X or x then (p, q) will be x. If (r, s) is neither X nor x then (p, q) will be '0'.

→ for $(A,B),r=\delta(A,a)=B,\ \ s=\delta(B,a)=C.$

It can be check from matrix in Fig. 3 that (B,C) is × so (A,B) will be x.

→ for $(A,E),r=\delta(A,a)=B,s=\delta(E,a)=F$

(B,F) is also × so (A,E) will be x.

→ for $(A,H),r=\delta(A,H)=B,s=\delta(H,a)=H$

(B,H) is not × but it is x so (A,H) will be x.

→ for $(A,I),r=\delta(A,a)=B,s=\delta(I,a)=I$

(B,I) is not × but it is x so (A,I) will be x.

→ for $(B,H),r=\delta(B,a)=C,s=\delta(H,a)=H$

(C,H) is × so (B,H) will be x.

→ for $(B,I),r=\delta(B,a)=C,s=\delta(I,a)=I$

(C,I) is × so (B,I) will be x.

→ for $(E,H),r=\delta(E,a)=F,s=\delta(H,a)=H$

(F,H) is × so (E,H) will be *x*.

→ for $(E,I),r=\delta(E,a)=F,s=\delta(I,a)=I$

(F,I) is × so (E,I) will be x.

→ for $(H,I),r=\delta(H,a)=H,s=\delta(I,a)=I$

(H,I) is neither × nor x so (H,I) will be "0".

Now consider $F=\{C,D,F,G\}$

Ordered pair of F × F $=\{(C,D),(C,F),(C,G),(D,F),(D,G),(F,G)\}$

→ for $(C,D),r=\delta(C,a)=H,s=\delta(D,a)=I$

(H,I) is neither × nor x so (C,D) will be "0".

→ for $(C,F),r=\delta(C,a)=H,s=\delta(F,a)=H$

(H,H) is dash so (C,F) will be "0".

$\rightarrow$ for $(C,G), r=\delta(C,a)=H, s=\delta(G,a)=H$

(H,H) is dash so (C,G) is "0".

$\rightarrow$ for $(D,F), r=\delta(D,a)=I, s=\delta(F,a)=H$

(I,H) is dash so check (H,I), it is also neither $\times$ nor x so (D,F) is "0" entry.

$\rightarrow$ for $(D,G), r=\delta(D,a)=I, s=\delta(G,a)=H$

(D,G) is also "0" entry.

$\rightarrow$ for $(F,G), r=\delta(F,a)=H, s=\delta(G,a)=H$

(H,H) is dash so (F,G) will be "0" entry.

Rest of the entries in upper triangle will be "0". Now fill the "x" and "0" in matrix given in Fig. 3.

	A	B	C	D	E	F	G	H	I
A	–	x	$\times$	$\times$	x	$\times$	$\times$	x	x
B	–	–	$\times$	$\times$	0	$\times$	$\times$	x	x
C	–	–	–	0	$\times$	0	0	$\times$	$\times$
D	–	–	–	–	$\times$	0	0	$\times$	$\times$
E	–	–	–	–	–	$\times$	$\times$	x	x
F	–	–	–	–	–	–	0	$\times$	$\times$
G	–	–	–	–	–	–	–	$\times$	$\times$
H	–	–	–	–	–	–	–	–	0
I	–	–	–	–	–	–	–	–	–

Fig: 4

The "0" at (B,E) means (B,E) is a state in minimum machine.

So if resulting minimum machine is M'= $\left(Q',\sum', \delta', A', F'\right)$

$Q'=(\{A\},\{B,E\},\{C,D,F,G\},\{H,I\})$

$F'=(\{C,D,F,G\})$

$A'=A$

Now consider Fig. 1 (given DFA) and draw the transition table for minimum machine as

$\delta'(\{A\},a)=\{B,E\}$

Since $\delta(A,a)=B$ in given DFA, B is in $\{B,E\}$ state of minimized machinery on the same way, the transition table for M' is

$\delta'/\sum$	a	b
$\rightarrow\{A\}$	$\{B,E\}$	$\{B,E\}$
$\{B,E\}$	$\{C,D,F,G\}$	$\{C,D,F,G\}$
$*\{C,D,F,G\}$	$\{H,I\}$	$\{H,I\}$
$\{H,I\}$	$\{H,I\}$	$\{H,I\}$

So minimized machine is

$\{A\}$ – M

$\{B,E\}$ – N

$\{C,D,F,G\}$ – P

$\{H,I\}$ – R

Fig: 5 Minimized DFA

16. Using Pumping lemma prove that $L=\left\{\{a^{i^3}\,\middle|\, i\geq 1\right\}$ is not regular. Construct Finite Automata equivalent to the Regular Expression.

L=ab(aa+bb)(a+b)*b. **[WBUT 2014]**

Answer:

Suppose L in regular and we get a contradiction. Let n be the number of states in FA accepting L.

Let $w=a^{n^3}$. Then $|w|=n^3>n$ by pumping lemma we can write

$w=xyz$ with $|xy|\leq n$ and $|y|>0$

Consider $xy^3z\cdot|xy^3z|=|x|+3|y|+|z|>|x|+|y|+|z|$

as $|y|>0$. This means $n^3=|xyz|=|x|+|y|+|z|<|xy^3z|$

as $|xy|\leq n, |y|\leq n$. Therefore

$$|xy^3z|=|x|+3|y|+|z|\leq n^3+n$$

i.e., $n^3<|xy^3z|\leq n^3+n\leq n^3+3n^2+3n+1$

Hence, $|xy^3z|$ strictly lies between n^3 and $(n+1)^3$ but it is not equal to any one of them.

Thus $|xy^3z|$ in not perfect case and so $xy^3z\notin L$. But by pumping lemma $xy^3z\in L$. This is a contradiction.

17. Convert the following Mealy machine to equivalent Moore machine.

Present State	I/P=0		I/P=1	
	Next State	O/P	Next State	O/P
q0	q1	1	q2	1
q1	q3	0	q0	1
q2	q4	0	q3	1
q3	q1	0	q4	0
q4	q2	1	q4	0

[WBUT 2014, 2017, 2019]

Answer:

State	Output
q_0	1
q_1	0, 1
q_2	0, 1
q_3	0
q_4	0, 1

State Redesignated

State	Output	Redesignated
q_0	1	q_0
q_1	0, 1	q_{10} , q_{11}
q_2	0, 1	q_{20} , q_{21}
q_3	0	q_3
q_4	0, 1	q_{40} , q_{41}

Moore Machine Table

Present State	Next State		Output
	$x=0$	$x=1$	
q_0	q_{11}	q_{21}	1
q_{10}	q_3	q_0	0
q_{11}	q_3	q_0	1
q_{20}	q_{40}	q_3	0
q_{21}	q_{40}	q_3	1
q_3	q_{10}	q_{40}	0
q_{40}	q_{21}	q_{40}	0
q_{41}	q_{21}	q_{40}	1

18. Find equivalent partitions and minimize the following Finite State Machine.

PS	NS, z	
	x=0	x=1
A	B, 0	H, 1
B	C, 0	G, 1
C	B, 0	F, 1
D	F, 1	C, 1
E	B, 1	C, 1
F	B, 1	B, 1
G	C, 1	B, 1
H	D, 1	A, 1

[WBUT 2014]

Minimize the following incompletely specified machine. **[WBUT 2014, 2017]**

PS	NS, z		
	I_1	I_2	I_3
A	A, 1	D, _	C, _
B	A, _	D, _	E, _
C	E, 0	A, 1	_, _
D	E, _	A, 1	_, _
E	E, 0	_, _	C, _

Answer:

1st Part:

Equivalence partitions are

P_0 = (ABCDEFGH)
P_1 = ((ABC) (DEFGH))
P_2 = ((ABC) ((DH) (EFG)))
P_3 = (((A) (BC)) (((D) (H)) (EFG)))
P_4 = (((A) (BC)) (((D) (H)) (EFG)))

Minimization: We know that equivalent partition is unique.
So, P_3 = (((A) (BC)) (((D) (H)) (EFG))) is the unique combination. Here, every single set represents one state of the minimized machine.
Let us rename these partitions for simplification.
Rename (A) as S_1, (BC) as S_2, (D) as S_3, (H) as S_4 and (EFG) as S_5(A) with input 0 goes to (B) and so there will be a transaction from S_1 to S_2 with input 0. (A) with input 1 goes to (H) and so there will be a transaction from S_1 to S_4 with input 1. (BC) with input 0 goes to (BC) for input 0. There will be a transaction from S_2 to S_2 for input 0. (BC) with input 1 goes to (FG). There will be a transaction from S_2 to S_5 for input 1.
By this process, the whole table of the minimized machine is constructed.

The minimized machine becomes

Present State	Next State, z	
	X = 0	X = 1
S_1(A)	S_2, 0	S_4, 1
S_2(BC)	S_2, 0	S_5, 1
S_3(D)	S_5, 1	S_2, 1
S_4(H)	S_3, 1	S_1, 1
S_5(EFG)	S_2, 1	S_2, 1

2nd Part:

$$P_0 = (ABCDE)$$

$$P_1 = (AB)(CDE)$$

This is the final partition.

Hence minimized machine is:

PS	NS, Z		
	I_1	I_2	I_3
S_1 = (AB)	S_1, 1	S_2, —	S_2, —
S_2 = (CDE)	S_2, 0	S_1, 1	S_2, —

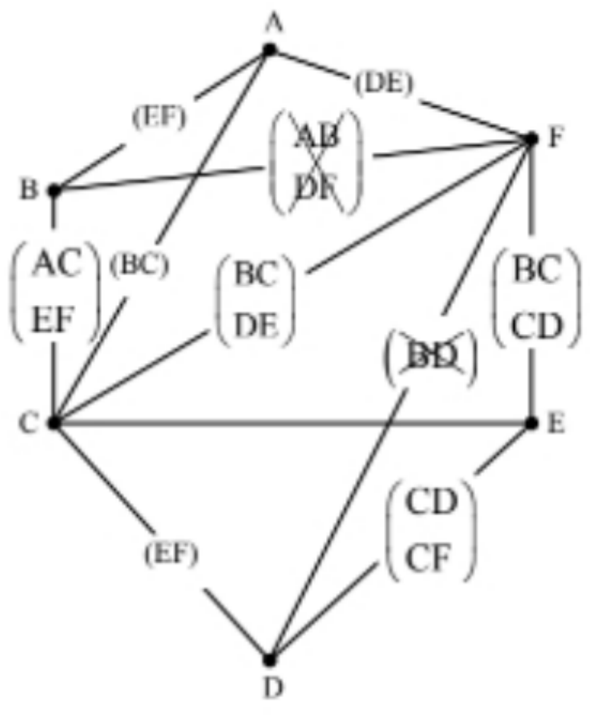
19. Consider the following state table

Present state	Next state,	output
	I1	I2
A	E, 0	B, 0
B	F, 0	A, 0
C	E, -	C, 0
D	F, 1	D, 0
E	C, 1	C, 0
F	D, -	B, 0

a) Draw the merger graph. **b) Draw the merger table.**
c) Draw the compatibility graph.
d) Find the minimal machine which covers that machine. **[WBUT 2015]**

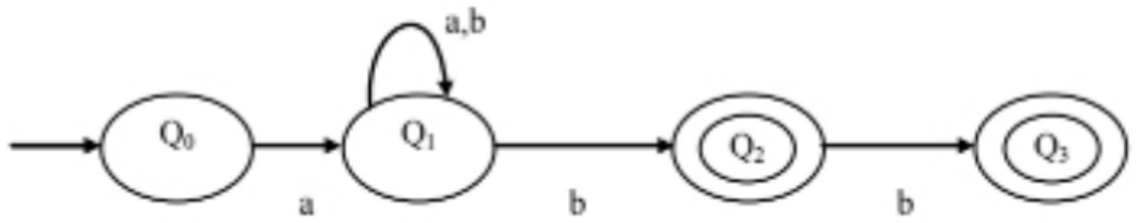
Answer:

a)



b), c) & d) ***Refer to Question No. 10(b) of Long Answer Type Questions.***

20. Consider the N.F.A given by the following diagram **[WBUT 2015]**

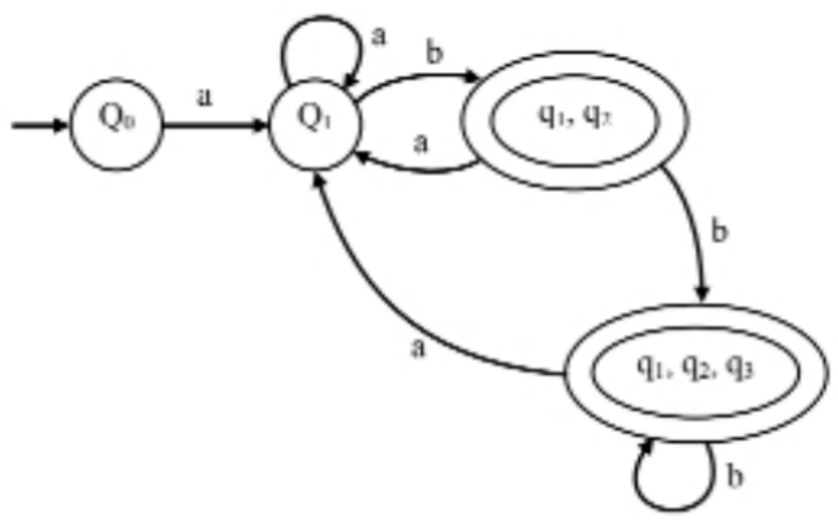


Find the equivalent D.F.A without e-transition.

Answer:

State	a	b
Q_0	$\{Q_1\}$	ϕ
Q_1	$\{Q_1\}$	$\{Q_1, Q_2\}$
$\{Q_1, Q_2\}$	$\{Q_1\}$	$\{Q_1, Q_2, Q_3\}$
$\{Q_1, Q_2, Q_3\}$	$\{Q_1\}$	$\{Q_1, Q_2, Q_3\}$

So the equivalent D.F.A. is



21. Using Pumping lemma prove that $L = \{a^n b^n \mid n \geq 1\}$ **is not regular.** **[WBUT 2017]**

Answer:

At first, we assume that **L** is regular and n is the number of states.

Let $w = a^i b^i$. Thus $|w| = 2i \geq i$.

By pumping lemma, let $w = xyz$, where $|xy| \leq i$.

Let $x = a^p$, $y = a^q$, and $z = a^r b^i$, where $p + q + r = n$, $p \neq 0$, $q \neq 0$, $r \neq 0$. Thus $|y| \neq 0$.

Let $k = 2$. Then $xy^2z = a^p a^{2q} a^r b^i$.

Number of $a = (p + 2q + r) = (p + q + r) + q = n + q$

Hence, $xy^2z = a^{i+q} b^i$. Since $q \neq 0$, xy^2z is not of the form $a^i b^i$.

Thus, xy^2z is not in L. Hence L is not regular.

22. State and prove Arden's theorem in regular expression. **[WBUT 2017]**

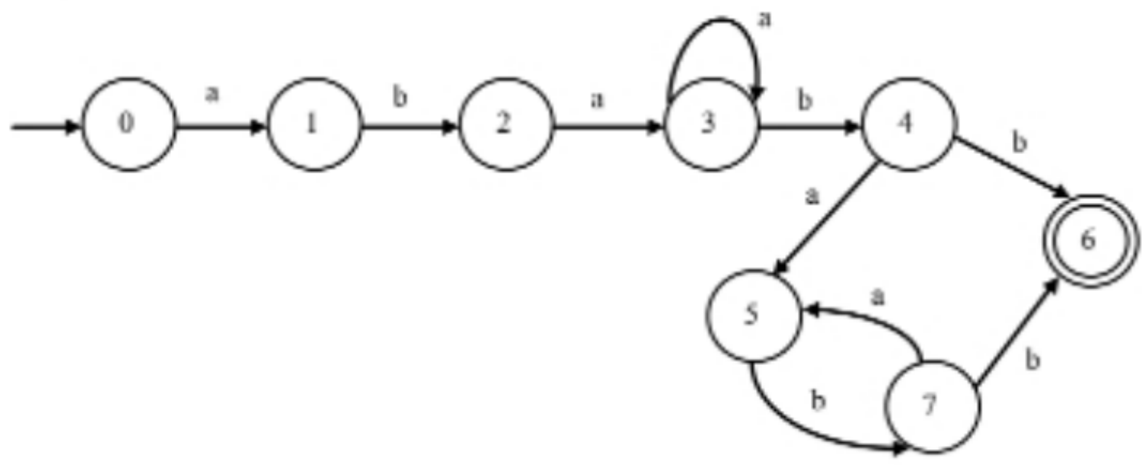
OR,

State and prove Erden's Theorem. **[WBUT 2018]**

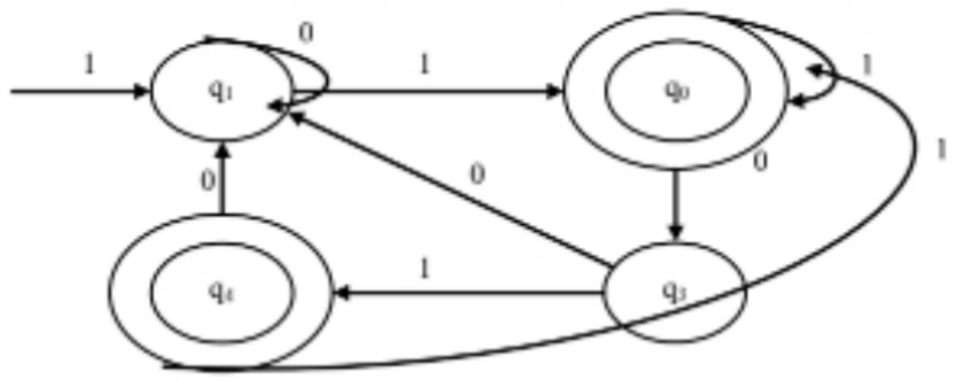
Answer:
In order to find out a regular expression of a Finite Automaton, we use Arden's Theorem along with the properties of regular expressions.
Statement −
Let **P** and **Q** be two regular expressions.
If **P** does not contain null string, then **R = Q + RP** has a unique solution that is **R = QP***
Proof −
R = Q + (Q + RP)P [After putting the value R = Q + RP]
= Q + QP + RPP
When we put the value of **R** recursively again and again, we get the following equation −
$R = Q + QP + QP^2 + QP^3 \ldots..$
$R = Q\,(\varepsilon + P + P^2 + P^3 + \ldots.)$
R = QP* [As P* represents (ε + P + P2 + P3 + ….)]
Hence, proved.

23. Construct Finite Automata equivalent to the Regular Expression
$L = ab(a+b)(ab)^*b$. **[WBUT 2017]**

Answer:



24. Find the regular expression corresponding to the following figure: [WBUT 2018]



Answer:
Using Arden's theorem —
$$R = Q + RP,\ R = QP^*$$
∴ Here,
$$q_1 = \varepsilon + q_1 0 + q_4 0 + q_3 0$$
$$q_0 = q_0 1 + q_4 1 + q_1 1$$
$$q_3 = q_0 0$$

$$q_4 = q_3 1$$

$\therefore \quad q_1 = \varepsilon + q_4 0 + q_3 0 + q_1$

$\therefore \quad q_0 = q_0 1 + q_4 1 + q_1 1$

$\Rightarrow \quad q_0 = q_0 1 + q_3 11 + q_1 1$

$\Rightarrow \quad q_0 = q_0 1 + q_0 011 + q_1 1$

$\Rightarrow \quad q_0 = q_1 1 + q_0 (1 + 011)$

$\Rightarrow \quad q_0 = q_1 1 (1 + 011)^*$

$\therefore \quad q_1 = \varepsilon + q_4 0 + q_3 0 + q_1$

$\Rightarrow \quad q_1 = \varepsilon + q_3 10 + q_3 0 + q_1$

$\Rightarrow \quad q_1 = \varepsilon + q_0 010 + q_0 00 + q_1$

$\Rightarrow \quad q_1 = \varepsilon + q_1 1(1 + 011)^* 010 + q_1 1(1 + 011)^* 00 + q_1$

$\Rightarrow \quad q_1 = \varepsilon + q_1 \left(1 \cdot (1 + 011)^* 010 + 1(1 + 011)^* 00 + \varepsilon\right)$

$\Rightarrow \quad q_1 = \varepsilon \cdot \left(1(1 + 011)^* 010 + 1(1 + 011)^* 00 + \varepsilon\right)^*$

$\Rightarrow \quad q_1 = \left(1 + (1 + 011)^* 010 + 1(1 + 011)^* 00\right)^*$

$\Rightarrow \quad q_4 = q_3 1 = q_0 01 = q_1 1(1 + 011)^* 01$

$$= \left(1(1 + 011)^* 010 + 1(1 + 011)^* 00\right)^* 1(1 + 011)^* 01$$

$\therefore$ Final regular expression:

$$(q_4 + q_0) = \left(\left(1 + (1 + 011)^* 010 + 1(1 + 011)^* 00\right)^* 1(1 + 011)^*\right)(\varepsilon + 01)$$

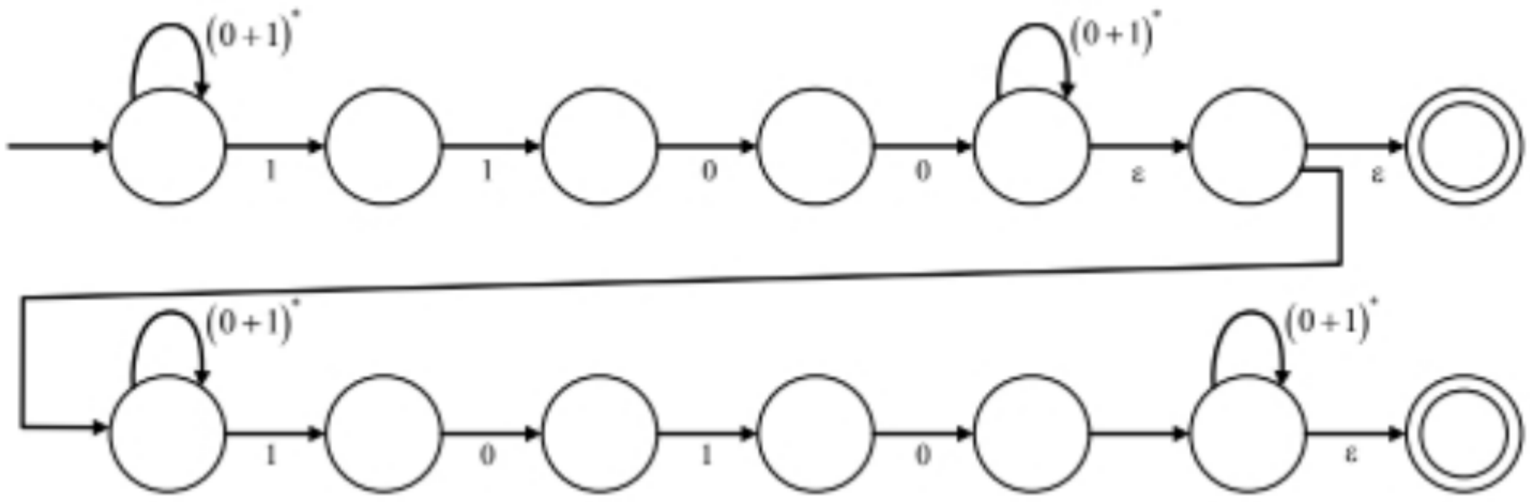
25. Design an NFA which accepts set of all binary strings containing 1100 or 1010 as substrings. **[WBUT 2018]**

Answer:

$\therefore$ A NFA over $\Sigma = \{0, 1\}^*$ with 1100 or 1010 as substring.

$\therefore$ Regular expression:

$$(0+1)^* 1100 (0+1)^* + (0+1)^* 1010 (0+1)^*$$



M/C: $(0+1)^* 1100 (0+1)^* + (0+1)^* 1010 (0+1)^*$

26. Construct a minimum state automata equivalent to a given automata M which transition table is given below: **[WBUT 2018]**

State	input	
	a	b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
q_6	q_1	q_3

Answer:

State	input	
	a	b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
q_6	q_1	q_3

$\therefore$ Partition algorithm is applied —

$$p_0 = (q_0\ q_1\ q_2\ q_3\ q_4\ q_5\ q_6)$$

$$p_1 = p_0$$

As, here not any state has same outputs and also no same state transition.
So, the given m/c is itself the minimized one.

27. Construct the merger table, merger graph, compatibility graph and minimal machine for the following Machine: **[WBUT 2018]**

PS	NS, z	
	I_1	I_2
A	E, 0	B, 0
B	F, 0	A, 0
C	E, –	C, 0
D	F, 1	D, 0

E	C, 1	C, 0
F	D, –	B, 0

Answer:

Refer to Question No. 8(a), (b), (c) & (d) of Long Answer Type Questions.

28. Construct testing table, testing graph for the following machine and test whether it has finite memory or Not, if yes then find the order. [WBUT 2018]

PS	NS, z	
	$x=0$	$x=1$
A	B, 0	D, 0
B	C, 0	C, 0
C	D, 0	A, 0
D	D, 0	A, 1

Answer:

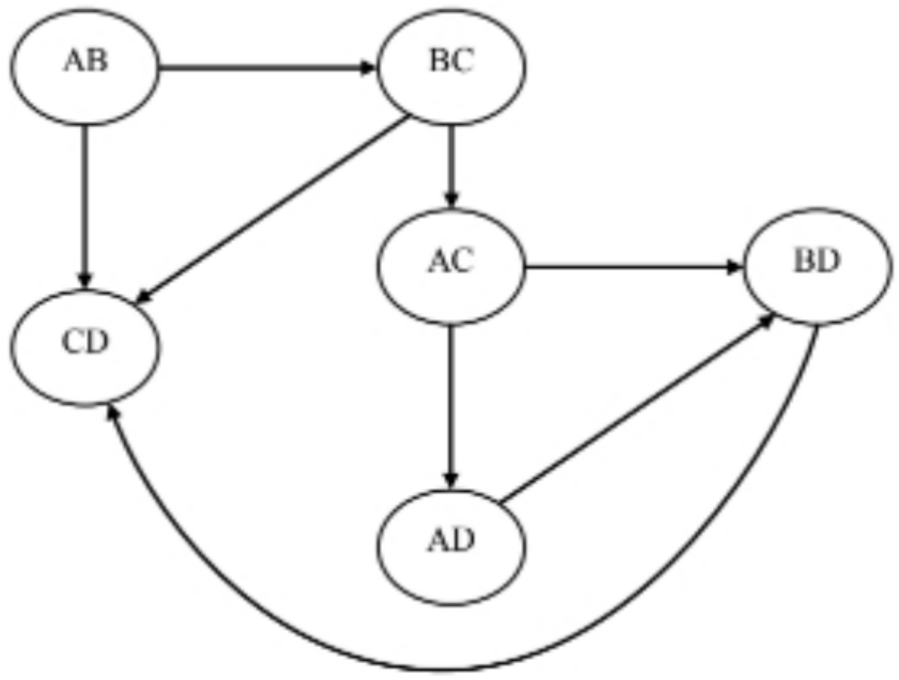
PS	NS, z	
	$x=0$	$x=1$
A	B, 0	D, 0
B	C, 0	C, 0
C	D, 0	A, 0
D	D, 0	A, 1

Testing Table:

PS	0/0	0/1	1/0	1/1
A	B	–	D	–
B	C	–	C	–
C	D	–	A	–
D	D	–	–	A
(AB)	(BC)	–	(CD)	–
(AC)	(BD)	–	(AD)	–
(AD)	(BD)	–	–	–
(BC)	(CD)	–	(AC)	–
(BD)	(CD)	–	–	–
(CD)	(DD)	–	–	–

(discarded)

Testing Graph:



∴ Here, in this testing graph, ∃ no loop so, we can say it is a finite memory machine.

∴ Order (μ) = Maximum length path $(\ell)+1$

∴ Here,

$$(AB \to BC \to AC \to AD \to BD \to CD)$$

$$\uparrow$$

max. path length is 6

∴ $\mu = 6+1=7$

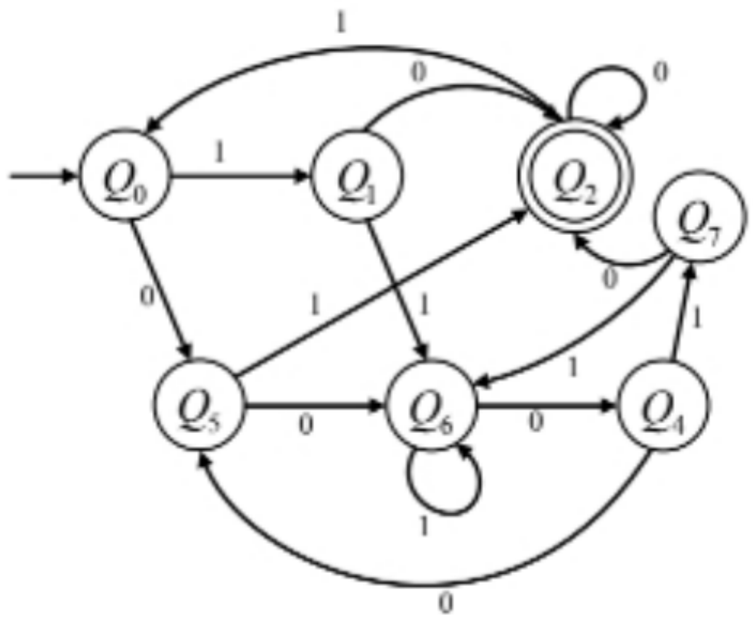
29. Construct the minimum state automation equivalent to given automata M defined below: **[WBUT 2019]**

Present State	Next State	
	$a=0$	$a=1$
$\to Q_0$	Q_5	Q_1
Q_1	Q_2	Q_6
Q_2	Q_2	Q_0
Q_4	Q_5	Q_7
Q_5	Q_6	Q_2
Q_6	Q_4	Q_6
Q_7	Q_2	Q_6

where Q_2 in the final state.

Answer:

Present State	Next State	
	$a=0$	$a=1$
$\rightarrow Q_0$	Q_5	Q_1
Q_1	Q_2	Q_6
(Q_2)	Q_2	Q_0
Q_4	Q_5	Q_7
Q_5	Q_6	Q_2
Q_6	Q_4	Q_6
Q_7	Q_2	Q_6



CONTEXT FREE LANGUAGES

☞ Chapter at a Glance

Context–Free Languages (CFL's) played a central in role natural languages since the 1950's, and in compilers since the 1960's.

Context-Free Grammars (CFG's) are the basis of BNF-syntax. Today CFL's are increasingly important for XML and their DTD's.

The Language of Grammar:

If $G(V,T,P,S)$ is a CFG, then the language of G is $L(G)=\left\{\omega \in T^* : S \overset{*}{\Rightarrow}_G \omega\right\}$ i.e. the set of strings over T^* derivable from the start symbol. If G is a CFG, we call $L(G)$ a context -free language.

Parse Trees: Parse trees are an alternative representation to derivations and recursive inferences. There can be several parse trees for the same string.
Ideally there should be only be only one parse tree for each string. i.e. the language should be unambiguous.
Unfortunately, we cannot always remove the ambiguity.

Ambiguity in Grammar/ Languages:

Let G =(V,T,P,S) be a CFG. We say that G is ambiguous is there is a string in T* that has more than one parse tree.
If every string in L(G) has at most one parse tree, G is said to be unambiguous.

Chomsky Normal Form:

We want to show that every *CFL* (without ε) is generated by a *CFG* where all productions are of the form

$$A \to BC, or A \to a\,.$$

Where and are variables, and is a Terminal. This is called CNF, and to get there we have to.

Eliminate useless symbols, those that do not appear in any derivation $S \overset{*}{\Rightarrow} \omega,$ for start symbol S and terminal $\omega.$

Eliminate ε *productions*, that is, productions of

The form $A \to \varepsilon$.

Eliminate *unit productions*, that is, productions of the for $A \to B$, where A and B are variables.

Greibach Normal Form:

A Context Free Grammar is said to be in Greibach Normal Form if each rule in the grammar has one of the following forms:

1. $A \to aA_1A_2 \ldots A_n$
2. $A \to a$ WHERE $A; A_i \in V_N, i = 1, 2 \ldots\ldots\ldots n \text{ and } a \in \sum$

If ε belongs to the language of the grammar, the rule $A \to \varepsilon$ (where S is the start symbol) is also permitted.

Greibach's Theorem: Every Context Free Language can be gener- ated by a Context Free Grammar which is in Greibach Normal Form GNF.

The above theorem implies that given a grammar G (which generates the language L(G), the grammar can be converted to an equivalent grammar G' which is in GNF such that $L(G') = L(G)$.

Multiple Choice Type Questions

1. A grammar in CNF may contain productions like **[WBUT 2007]**

a) $A \to B$ **b)** $A \to BC$ **c)** $A \to aB$ **d)** $A \to aBCD$

Answer: (b)

2. $L = \{a^n b^n c^n, \text{ where } n \in 1\}$ **is** **[WBUT 2009]**

a) regular **b) context free but not regular**

c) context sensitive but not context free **d) none of these**

Answer: (c)

3. Which of the following productions is in CNF? **[WBUT 2009]**

a) $S \to aA$ **b)** $SA \to AS$ **c)** $S \to AB$ **d) All of these**

Answer: (c)

4. Which is more suitable for an Ambiguous Grammar? **[WBUT 2009]**

a) All ambiguities can be removed

b) Ambiguity can be removed by setting priority

c) Only inherent ambiguity can be removed

d) There is no suitable rule for removing ambiguity

Answer: (c)

5. The class of CFG is not closed under **[WBUT 2011]**

a) concatenation **b) intersection**

c) union **d) repeated concatenation**

Answer: (b)

6. Consider the CFG **[WBUT 2011]**

$$X \to XY$$
$$X \to zX / bX / a$$
$$Y \to Ya / Yb / b$$

Any string of terminals, which can be generated by the CFG

a) has at least one b
b) ends with a
c) has no consecutive a's or b's
d) has at least 2 a's

Answer: (d)

7. A grammar that produces more than one parse tree for some sentence is said to be **[WBUT 2011, 2019]**

a) contiguous b) ambiguous c) unambiguous d) regular

Answer: (b)

8. A CFG, $S \to aS / bS / a / b$, is equivalent to **[WBUT 2011]**

a) $(a+b)^+$ b) $(a+b)(a+b)^*$ c) $(a+b)^*(a+b)$ d) all of these

Answer: (b)

9. $(P+Q)^* = ?$ **[WBUT 2012]**

a) $(P^* + Q^*)$ b) $P^* + Q^*$ c) $(P^* Q^*)^*$ d) both (a) and (c)

Answer: (d)

10. Which of the following production is in CNF? **[WBUT 2013]**

a) S $\to$ aA b) SA $\to$ AS c) S $\to$ AB d) All of these

Answer: (c)

11. Consider the following language: $L = \{a^n b^n c^n d^n \mid n \geq 1\}$

L is **[WBUT 2013]**

a) CFL but not regular
b) CSL but not CFL
c) Regular
d) Type 0 language but not type 1

Answer: (b)

12. Consider the grammar S $\to$ aSAb/$\in$

A $\to$ bA/$\in$

The grammar generates the strings of the form $a^i b^j$ for some i, j≥0. What is the conditions of the values of i and j? **[WBUT 2014, 2015]**

a) i = j b) j ≤ 2i c) j ≥ 2i d) i ≤ j

Answer: (d)

13. Which of the following common in both CNF & GNF? **[WBUT 2014, 2015, 2017]**

a) (NT) → (Single T) (String of NT) b) (NT) → (Single of exactly two NT)

c) (NT) → (String of NT) d) (NT) → (Single T)

Answer: (d)

14. Consider the languages:

$$L_1 = \left\{ \{WW^R \middle| W \in \{0,1\}^*\right\}$$

$$L_2 = \left\{ \{W\#W^R \middle| W \in \{0,1\}^*\right\}$$, **where # is a special symbol**

$$L_3 = \left\{ \{WW \middle| W \in \{0,1\}^*\right\}$$

Which one of the following is true? **[WBUT 2014, 2015, 2017]**

a) L_1 is a deterministic CFL b) L_2 is a deterministic CFL

c) L_3 is a CFL, but not a deterministic CFL

d) L_3 is a deterministic CFL

Answer: (b)

15. A context free grammar is not closed under **[WBUT 2016]**

a) Product b) Union

c) Complementation d) Kleen star

Answer: (c)

16. Context sensitive grammar can be recognized by **[WBUT 2016]**

a) deterministic push down machine

b) non-deterministic push down machine

c) FSM

d) linearly bounded memory machine

Answer: (d)

17. Which of the following is most general phase structured grammar?

a) Regular b) Context sensitive **[WBUT 2016]**

c) PDA d) none of these

Answer: (b)

18. CFL is _____ language. **[WBUT 2018]**

a) Type 0 b) Type 1 c) Type 2 d) Type 3

Answer: (c)

Short Answer Type Questions

1. Convert the following context free grammar into an equivalent grammar in CNF:

$$S \rightarrow aAbB$$

$$A \rightarrow abAB / aAA / a$$ **[WBUT 2006, 2007, 2010, 2019]**

$$B \rightarrow bBaA / aBB / b$$

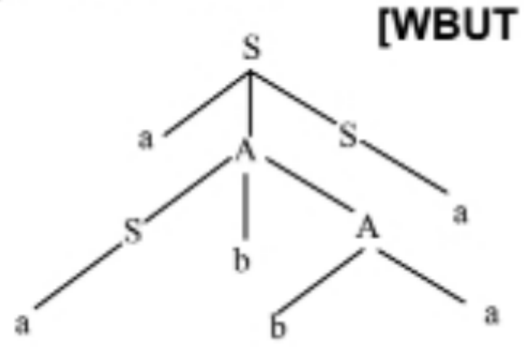
Answer:

Let us assume that a and b may be represented as C and D respectively.

So, the products are as follows:-
S→CABD
A→CDAB|CAA|a
B→DBCA|CBB|b
C→a
D→b
Let us assume that AA be E, BB be F, CA be G and DB be H. So the productions are:-
S→GH
A→GH
B→GH
C→a
D→b
E→AA
F→BB
G→CA
H→BD

2. a) What do you mean by a sub-tree of a derivation tree? [WBUT 2008]
Answer:
A subtree of a derivation tree is a *partial derivation tree* such that, for any node of the subtree, either all of its children are also in the subtree, or none of them are.

b) Consider G whose productions are S→aAS/a, A→SbA/SS/ba. Show that S→aabbaa by constructing a derivation tree, by right most derivation, whose yield is aabbaa. [WBUT 2008]
Answer:
S→aAS
→aSbAS
→aabAS
→aabbaS
→aabbaa (proved).



3. Reduce the following grammars to GNF: [WBUT 2008]
S→A0, A→0B, B→0A, B→1
Answer:
As it is already in GNF form, so there is no requirement for converting it into GNF.

4. Write the CFG for the language [WBUT 2008, 2012]
$L=\{0^i 1^j 2^k \mid i=j \text{ or } j=k\}$.
Answer:
S→AB|B
A →0A|0
B→1B2|12

5. Let *G* be a grammar **[WBUT 2009, 2010]**

$$s \to 0B \mid 1A,\ A \to 0 \mid 0S \mid 1AA,\ B \to 1 \mid 1S \mid 0BB\ .$$

For the string 00110101, find

i) leftmost derivation

ii) rightmost derivation

iii) derivation tree

OR,

Convert the following Context-free grammar into an equivalent grammar in CNF

$S \to 1A / 0B$

$A \to 1AA / 0S / 0$

$B \to 0BB / 1S / 1$

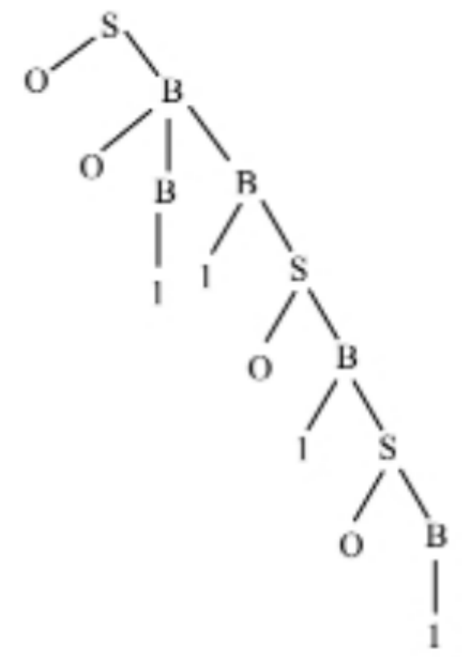
[WBUT 2011]

Answer:

i) $S \Rightarrow 0\underline{B} \Rightarrow 00\underline{B}B \Rightarrow 001\underline{B} \Rightarrow 0011\underline{S} \Rightarrow 00110\underline{B} \Rightarrow 001101\underline{S}$
$\Rightarrow 0011010\underline{B} \Rightarrow 00110101$

ii) $S \Rightarrow 0\underline{B} \Rightarrow 00B\underline{B} \Rightarrow 00B1\underline{S} \Rightarrow 00B10\underline{B} \Rightarrow 00B101\underline{S}$
$\Rightarrow 00B1010\underline{B} \Rightarrow 00\underline{B}10101 \Rightarrow 00110101$

iii)



6. Let G be the grammar **[WBUT 2011]**

$$S \to aB / ba,\ A \to a / aS / bAA,\ B \to b / bS / aBB$$

For the string $aaabbabbba$ **, find**

a) leftmost derivation

b) rightmost derivation

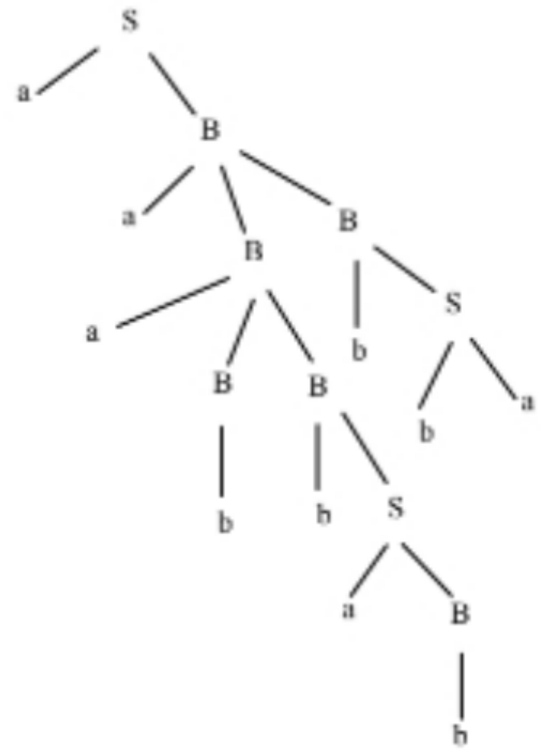
c) parse tree.

Answer:

a) $S \Rightarrow a\underline{B} \Rightarrow aa\underline{B}B \Rightarrow aaa\underline{B}BB \Rightarrow aaab\underline{B}B \Rightarrow aaabb\underline{S}B$
$\Rightarrow aaabba\underline{B}B \Rightarrow aaabbab\underline{B} \Rightarrow aaabbabb\underline{S} \Rightarrow aaabbabbba$

b) $S \Rightarrow a\underline{B} \Rightarrow aaB\underline{B} \Rightarrow aaBb\underline{S} \Rightarrow aa\underline{B}bba \Rightarrow aaaB\underline{B}bba$
$\Rightarrow aaaBb\underline{S}bba \Rightarrow aaaBba\underline{B}bba \Rightarrow aaa\underline{B}babbba \Rightarrow aaabbabbba$

c)



7. What is Griebach Normal Form (GNF) for Context Free grammar?
Convert the following grammar into GNF **[WBUT 2011, 2012, 2016]**

$S \rightarrow ABb / a$

$A \rightarrow aaA / B$

$B \rightarrow bAb$

Answer:

A Context Free Grammar is said to be in Greibach Normal Form if each rule in the grammar has one of the following forms:

1. $A \rightarrow aA_1A_2 \ldots A_n$
2. $A \rightarrow a$

where, $A; A_i \in V_N, i = 1,2.....n$ and $a \in \sum$

If ε belongs to the language of the grammar, the rule $A \rightarrow \varepsilon$ (where S is the start symbol) is also permitted.

Greibach's Theorem: Every Context Free Language can be gener- ated by a Context Free Grammar which is in Greibach Normal Form GNF.

The above theorem implies that given a grammar G (which generates the language $L(G)$, the grammar can be converted to an equivalent grammar G' which is in GNF such that $L(G') = L(G)$.

For Step-0, we need to add a rule $Y \rightarrow b$. So we start the main process of conversion of the grammar to GNF with the rules:

$S \rightarrow ABY | a$

$A \to aaA \mid B$

$B \to bBY$

$Y \to b$

There is no immediate left recurstion in the grammar.
We select the order $S, A, B, Y,$ Clearly there is no need for nonterminal substitution.
So, the final grammar in GNF is:

$Y \to b$

$B \to bBY$

$A \to aaA \mid bBY$

$S \to aaBY \mid bBY\ BY \mid a$

8. Find a GNF grammar equivalent to the following CFG: **[WBUT 2012]**

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 \mid b$

$A_3 \to A_1 A_2 \mid a$

Answer:
Given:

$$A_1 \to A_2 A_3$$

$$A_2 \to A_3 A_1 \mid b$$

$$A_3 \to A_1 A_2 \mid a$$

The grammar is already in CNF.
Round-1

$$A_1 \to A_2 A_3$$

$$A_2 \to A_1 A_2 A_1 \mid aA_1 \mid b$$

$$A_3 \to A_1 A_2 \mid a$$

Round-2

$$A_1 \to A_1 A_2 A_1 A_3 \mid aA_1 A_3 \mid bA_3$$

But this requires removal of left recursion

$$A_1 \to aA_1 A_3 Z \mid bA_3 Z \mid aA_1 A_3 \mid bA_3$$

$$Z \to A_1 A_2 A_1 A_3 Z \mid A_1 A_2 A_1 A_3$$

$$A_2 \to A_1 A_2 A_1 \mid aA_1 \mid b$$

$$A_3 \to A_1 A_2 \mid a$$

Round-3

$$A_1 \to aA_1A_3Z \mid bA_3Z \mid aA_1A_3 \mid bA_3$$

$$Z \to aA_1A_3ZA_2A_1A_3Z \mid$$

$$bA_3ZA_2A_1A_3Z \mid$$

$$aA_1A_3A_2A_1A_3Z \mid$$

$$bA_3A_2A_1A_3Z \mid$$

$$aA_1A_3ZA_1A_2A_1A_3 \mid$$

$$bA_3ZA_2A_1A_3 \mid$$

$$aA_1A_3A_2A_1A_3 \mid$$

$$bA_3A_2A_1A_3 \mid$$

$$A_2 \to aA_1A_3ZA_2A_1 \mid bA_3ZA_2A_1 \mid$$

$$aA_1A_3A_2A_1 \mid bA_3A_2A_1 \mid aA_1 \mid b$$

$$A_3 \to aA_1A_3ZA_2 \mid bA_3ZA \mid aA_1A_3A_2 \mid bA_3A_2 \mid a$$

The grammar is now in GNF.

9. Find the Context Free Grammar for the following language $L = \{a^n b^{2n} c^m : n, m >= 0\}$.

[WBUT 2013]

Answer:

Let us assume $CFG \Rightarrow G = (V_n, V_t, P, S)$

$V_n = \{S, A, B\}$, non terminals

$V_t = \{a, b, c\}$, terminals

P (production rule) is defined as follows:

$$S \to AB$$

$$A \to aAbb/\varepsilon$$

$$B \to cB/\varepsilon$$

10. Construct grammar of the following.

a) Construct the grammar for the language a1bmcn, where 1+m=n [WBUT 2014]

b) all even integers up to 998. [WBUT 2014, 2017]

Answer:

a) S -> aAc ; A->bBc|aAc| epsilon; B->bBc|epsilon

b) S->AB|AAB|0|2|4|6|8; A->0|1|2|3|4|5|6|7|8|9; B->0|2|4|6|8

11. Convert the following grammar into GNF.

S → AA/a

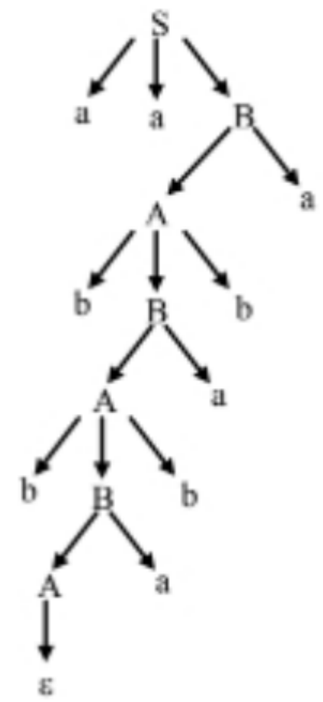
A → SS/b [WBUT 2014, 2017]

Answer:
S → AA|a
A→ SS | b
A→ AAS |aS | b
A→ aSZ | bZ | aS | b
Z→ASZ | epsilon
So the GNF
S → aSZA | bZA | aSA | bA | a
A → aSZ | bZ | aS | b
Z → aSZSZ | bZSZ | aSSZ | bSZ

12. a) What is parse tree? **[WBUT 2015]**
b) Consider the CFG
S- > aaB
A- > bBb / ε
B- > Aa
Find the Parse tree for the string aabbababa.
Answer:
a) A parse tree is an entity which represents the structure of the derivation of a terminal string from some non-terminal (not necessarily the start symbol). The definition is as in the book. Key features to define are the *root* $\in$ V and *yield* $\in \Sigma^*$ of each tree.

- For each $\sigma \in \Sigma$, there is a tree with root σ and no children; its yield is σ
- For each rule $A \rightarrow \varepsilon$, there is a tree with root A and one child ε; its yield is ε
- If $t_1, t_2, \ldots, t_n$ are parse trees with roots $r_1, r_2, \ldots, r_n$ and respective yields $y_1, y_2, \ldots, y_n$, and $A \rightarrow r_1 r_2 \ldots r_n$ is a production, then there is a parse tree with root A whose children are $t_1, t_2, \ldots, t_n$. Its root is A and its yield is $y_1 y_2 \ldots y_n$

b)

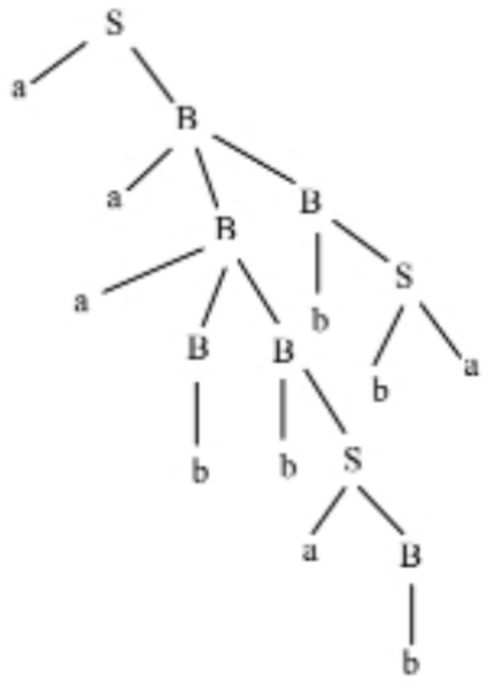


13. Let G be the grammar S → aB | ba, B → b | bS | aBB. [WBUT 2016]
For the string aaabbabbba find:
- **i) left most derivation**
- **ii) rightmost derivation**
- **iii) parse tree**

Answer:

i) S → a<u>B</u> → aa<u>B</u>B → aaaBBB→ aaabb<u>B</u> → aaabbaBB→ aaabbab<u>B</u>→ aaabbabb<u>S</u> → aaabbabbba

ii) $S \Rightarrow a\underline{B} \Rightarrow aaB\underline{B} \Rightarrow aaBb\underline{S} \Rightarrow aa\underline{B}bba \Rightarrow aaaB\underline{B}bba$
$\Rightarrow aaaBb\underline{S}bba \Rightarrow aaaBba\underline{B}bba \Rightarrow aaa\underline{B}babbba \Rightarrow aaabbabbba$



14. What do you mean by unit production? Remove unit productions from the grammar
$S \to AB,\ A \to a,\ B \to C,\ C \to D, D \to b.$ **[WBUT 2017]**

Answer:

1st Part:

A unit production is a production $A \to B$ where both A and B are non-terminals. Unit productions are redundant and hence should be removed.

Repeat the following steps while there is a unit production

1. Select a unit production $A \to B$, such that there exist a production $B \to \alpha$, where α is a terminal
2. For every non-unit production, $B \to \alpha$ repeat the following step
    1. Add production $A \to \alpha$ to the grammar
3. Eliminate $A \to B$ from the grammar

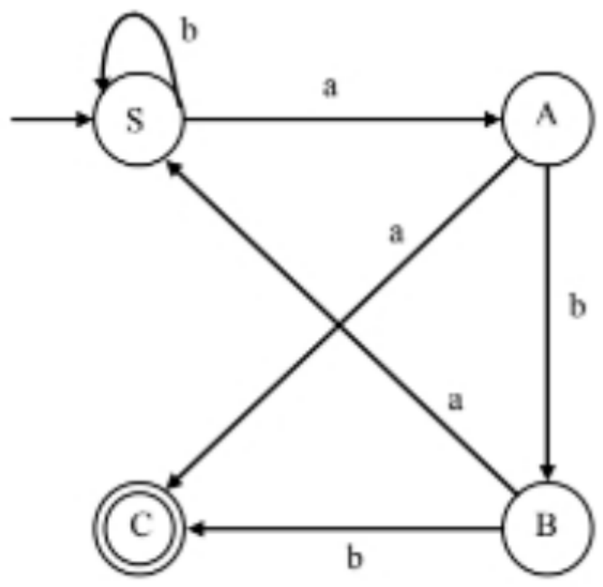
2nd Part:

There are 2 unit productions in the grammar –
$B \to C$
$C \to D$

For production $C \to D$ there is $D \to b$, so we add $C \to b$ and delete $C \to D$ from the grammar. Now we have $B \to C$, so we add a production $B \to b$ and remove $B \to C$ from the grammar.
Now we get the final grammar free of unit production as –
$S \to AB$
$A \to a$
$B \to b$
$C \to b$
Now we can see that C is an unreachable symbol and so to get a completely reduced grammar, we remove C from the CFG. Thus the final CFG is –
$S \to AB$
$A \to a$
$B \to b$

15. Construct grammar of the following:
For the language $a^n b^n$, where $n >= 0$. **[WBUT 2017]**
Answer:
L(G) = {ab, aabb, aaabbb, ...}
G = {N,T,P,S_t}
N= {S,A,B}
T= {a,b}
S_t = starting symbol = S
P = $S \to aA$, $A \to aA \mid bB$, $B \to bB \mid \varepsilon$

16. Define Left Linear and Right linear grammar. Construct grammar for the following FA: **[WBUT 2017]**



Answer:
1st Part:
Left Linear Grammar: In a grammar, if all productions are in the form –
$A \to B\alpha$ or $A \to \alpha$, where $A, B \in V_N$ and $\alpha \in \sum^*$, then the grammar is called Left Linear Grammar. Example: $A \to Aa \mid Bb \mid b$

Right Linear Grammar: In a grammar, if all productions are in the form – $A \to \alpha B$ or $A \to \alpha$, where $A, B \in V_N$ and $\alpha \in \sum^*$, then the grammar is called Right Linear Grammar. Example: $A \to aA \mid bB \mid b$

2nd Part:
The grammar **G** can be formally written as a 4-tuple (N, T, S_t, P) where –
N = {S,A,B,C}; N is a set of variables or non-terminal symbols
T = {a,b}; T is a set of terminal symbols
S_t = Starting Symbol (S)
P = Production rules for terminals and non-terminals $-S \to aA \mid bS$, $A \to aC \mid bB$, $B \to aS \mid bC$, $C \to \varepsilon$.

17. Remove the UNIT productions from the following grammar: [WBUT 2018]

$S \to Aa \mid B$
$B \to A \mid bb$
$A \to a \mid bc \mid B$

Answer:
Remove unit production,

$S \to Aa \mid B$
$B \to A \mid bb$
$A \to a \mid bc \mid B$

$\Rightarrow$ Here, three unit productions are present

$S \to B$, $A \to B$, $B \to A$

$\therefore$ $S \to Aa \mid A \mid bb$
$B \to a \mid bc \mid B \mid bb$
$A \to a \mid bc \mid A \mid bb$

So, $B \to a \mid bc \mid bb$
$S \to Aa \mid a \mid bc \mid bb$
$A \to a \mid bc \mid a \mid bc \mid bb$

We can remove the void production $B \to B$

$\therefore$ Final productions are $\to$

$B \to a \mid bc \mid bb$
$S \to Aa \mid a \mid bc \mid bb$
$A \to a \mid bc \mid bb$

Same productions can be removed. **(Ans.)**

18. Write the CFG for the language $L = \{0^i 1^j 2^k \mid i = j \text{ or } j = k\}$. **[WBUT 2019]**

Answer:

We can think of L as union of two languages

$$L = L_1 \cup L_2$$

where $L_1 = \{0^i 1^j 2^k \mid i = j\}$ and $L_2 = \{0^i 1^j 2^k, \ i = k\}$

We can create simple grammars for the separate languages and union them:

$$S \to S_1 \mid S_2$$

For L_1, We simply ensure that the number of 0's equal the number of 1's.

$$S_1 \to S_1 2 \mid A \mid \varepsilon$$
$$A \to 0A1 \mid \varepsilon$$

Similarly for ensuring that the number of 1's equals the number of 2's

$$S_2 \to 0S_2 \mid B \mid \varepsilon$$
$$B \to 1B2 \mid \varepsilon$$

This grammar is ambiguous. For $x = 0^n 1^n 2^n$, we may use either S_1 or S_2 to generate x.

Long Answer Type Questions

1. a) Let G be the grammar **[WBUT 2006, 2009]**

S → aB / bA
A → a / aS / bAA
B → b / bS / aBB

Find left most derivation for the string aaabbabbba.
a) leftmost derivation
b) rightmost derivation
c) parse tree.

Answer:

Let us number the rules

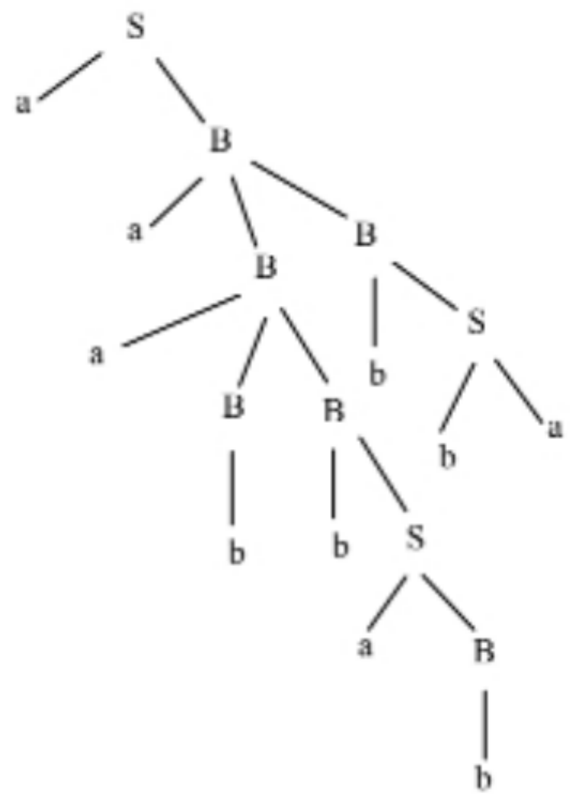
(1) $S \to aB$
(2) $S \to bA$
(3) $A \to a$
(4) $A \to aS$
(5) $A \to bAA$
(6) $B \to b$
(7) $B \to bS$
(8) $B \to aBB$

a) The left-most derivation (With ⇒ labeled with rule number) is:

$$\underset{(1)}{\Rightarrow} \underline{S} \underset{(8)}{\Rightarrow} a\,\underline{B} \underset{(8)}{\Rightarrow} aa\,\underline{B}\,B \underset{(6)}{\Rightarrow} aaa\,\underline{B}\,B\,B \underset{(7)}{\Rightarrow} aaa\,b\,\underline{B}\,B \underset{(1)}{\Rightarrow} aaa\,b\,b\,\underline{S}\,B \underset{(6)}{\Rightarrow} aaa\,b\,b\,a\,\underline{B}\,B \underset{(7)}{\Rightarrow} aaa\,bb\,a\,b\,\underline{B} \underset{(2)}{\Rightarrow} aaa\,bb\,a\,bb\,\underline{S} \underset{(3)}{\Rightarrow} aaa\,bb\,a\,bbb\,\underline{A} \Rightarrow aaa\,bb\,a\,bbb\,a$$

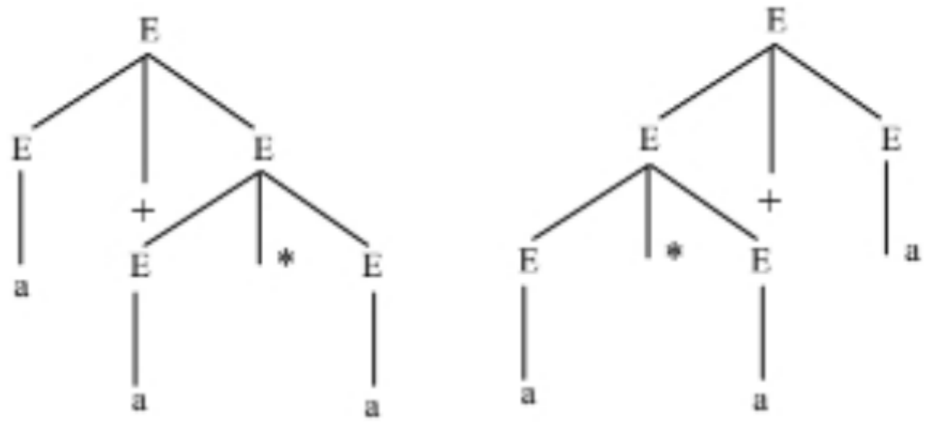
b) $S \Rightarrow a\underline{B} \Rightarrow aaB\underline{B} \Rightarrow aaBb\underline{S} \Rightarrow aa\underline{B}bba \Rightarrow aaaB\underline{B}bba$
$\Rightarrow aaaBb\underline{S}bba \Rightarrow aaaBba\underline{B}bba \Rightarrow aaa\underline{B}babbba \Rightarrow aaabbabbba$

c)



2. a) E→E+E|E*E|a. Prove that the CFG with this production rule is ambiguous. Remove the ambiguity from this grammar. **[WBUT 2008, 2019]**

Answer:



b) S→AB; A→a, B→C/b, C→D; D→E, E→a.
remove the unit production.
L={a^n b^n | n ≥ 0}.Find a CFG to generate L^2. **[WBUT 2008]**

Answer:
Here unit productions are:
B→C,
C→D,
D→E.
And original non-unit productions are:-
S→AB,
A→a,
B→b,
E→a.
Next right side of all unit production replace only terminal symbol so the productions are
S→AB,
A→a,
B→b,
E→a.
Then the complete productions are:-
S→AB,
A→a,
B→a | b,
E→a.
Suppose w= $a^n b^n \in L$.
Then awb= $a^{n+1}b^{n+1} \in L$
Hence required CFG is:
S→ awb | ε .

3. a) Convert grammars to Greibach Normal Form (GNF). **[WBUT 2009]**

i) $S \to aSa \mid aSb \mid \varepsilon$

ii) $S \to aSB \mid aSbS \mid \varepsilon$

Answer:
Strictly speaking, none of the given grammars can be converted to GNF since ε belongs to the languages of both the grammars. However, we continue by ignoring the $S \to \varepsilon$ productions and adding it to the converted grammar.

i) The converted grammar in GNF is

$S \to aSA$
$S \to aSB$
$S \to \varepsilon$
$A \to a$
$B \to b$

ii) The given grammar is incomplete since there are NO B productions given.

b) Find a reduced grammar equivalent to the grammar **[WBUT 2009, 2014, 2017]**
$S \to aAa, A \to bBB, B \to ab, C \to aB$.

Answer:

Clearly C is a useless symbol. Eliminating C, we get,

$$S \to aAa,\ A \to bBB,\ B \to ab$$

Now we replace all occurrences of B on rhs of production using $B \to ab$, getting:

$$S \to aAa,\ A \to babab$$

Now we similarly substitute A, getting

$$S \to a\,ba\,ba\,ba$$

which is the equivalent reduced grammar.

4. a) Construct CFG for the following: **[WBUT 2013, 2016]**

i) Palindrome for binary numbers

ii) $L = \{a^n b^n c^m d^m \mid m, n > 0\}$

iii) $L = \{a^n b^m \mid n \neq m\}$

b) Convert the following grammar to CNF

$S \to aA / B / C / a$

$A \to aB / E$

$B \to aA$

$C \to cCD$

$D \to abd$ **[WBUT 2013]**

c) Define non-generating and non-reachable symbols with example. **[WBUT 2013]**

Answer:

a) i) Consider $L_{pal} = \{w \in \Sigma^* : w = w^R\}$

For example otto $\in L_{pal}$, madamimadam $\in L_{pal}$.

In finish language e.g. saippuakauppias $\in L_{pal}$ ("soap- merchant")

Let $\Sigma = \{0,1\}$ and suppose L_{pal} were regular.

Let n be given by the pumping lemma. Then $0^n 10^n \in L_{pal}$ In reading 0^n the FA must make a loop. Omit the loop; contradiction.

Let's define L_{pal} inductively:

Basis: $\varepsilon, 0$, and 1 are palindromes.

Induction: If w is a palindrome, so are $0w0$ and $1w1$.

Circumscription: Nothing else is a palindrome.

CFG's is a formal mechanism for definitions such as the one for L_{pal}

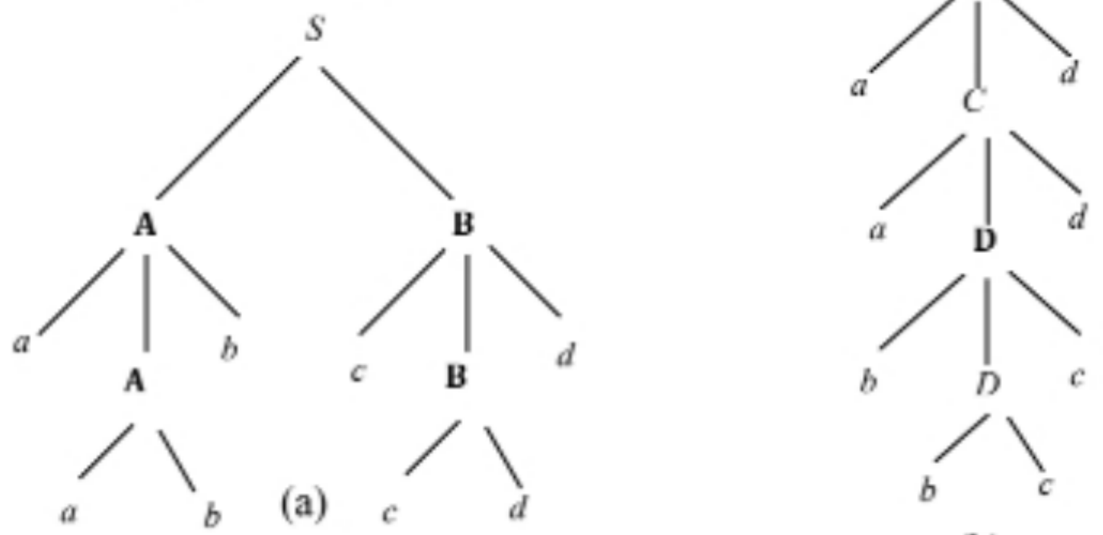
1. $P \to \varepsilon$
2. $P \to 0$
3. $P \to 1$
4. $P \to 0P0$
5. $P \to 1P1$

0 and 1 are terminals
P is a variable (or nonterminal, or syntactic category)
P is in this grammar also the start symbol.
1-5 are productions (or rules)

ii) A CFL L is inherently ambiguous if all grammars for L are ambiguous.
Example: Consider, $L = \{a^n b^n c^m d^m : n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n : n \geq 1, m \geq 1\}$.
A grammar for L is

$$\begin{aligned} S &\rightarrow AB \mid C \\ A &\rightarrow aAb \mid ab \\ B &\rightarrow cBd \mid cd \\ C &\rightarrow aCd \mid aDd \\ D &\rightarrow bDc \mid bc \end{aligned}$$

Let's look at parsing the string aabbccdd.



(b)

From this we see that there are two leftmost derivations:

$S \underset{lm}{\Rightarrow} AB \underset{lm}{\Rightarrow} aAbB \underset{lm}{\Rightarrow} aabbB \underset{lm}{\Rightarrow} aabbcBd \underset{lm}{\Rightarrow} aabbccdd$ And

$S \underset{lm}{\Rightarrow} C \underset{lm}{\Rightarrow} aCd \underset{lm}{\Rightarrow} aaDdd \underset{lm}{\Rightarrow} aabDcdd \underset{lm}{\Rightarrow} aabbccdd$

It can be shown that *every* grammar for L behaves like the one above. The language L is inherently ambiguous.

iii) CFG for the following is given below:

$$L = \{a^n b^m \mid n \neq m\}$$

If $n \neq m$, then two cases are possible
Case - 1

$$n > m$$

$$L_1 = \{a^n b^m : n > m\}$$

If G_1 be the CFG for L_1 language

$$G_A = \left(V_n^A, V_t^A, P^A, S^A\right)$$

$$V_n^A = \left\{S_1, A, S^A\right\}$$

$$V_t^A = \left\{a, b\right\}$$

Productions P^A are

$$S^A \to AS_1$$

$$S_1 \to aS_1b/\varepsilon$$

$$A \to aA/a$$

Case - 2

$$n < m$$

$$L_2 = \left\{0^n1^m : n < m\right\}$$

If G_2 be the CFG for L_2

$$G_B = \left(V_n^B, V_t^B, P^B, S^B\right)$$

$$V_n^B = \left\{S^B, S_2, B\right\}$$

$$V_t^B = \left\{a, b\right\}$$

P^B are :

$$S^B \to S_2B$$

$$S_2 \to aS_2b/\varepsilon$$

$$B \to bB/b$$

$\therefore$ By combining $\left(G_A \,\&\, G_B\right)$ we can write CFG for the above.

$S \to S^A/S^B$ where S is the start symbol.

b) $S \to aA \,/\, B \,/\, C \,/\, a$

$A \to aB \,/\, E$

$B \to aA$

$C \to cCD$

$D \to abd$

CNF $\to$ $S \to C_1A \,/\, C_2B \,/\, C_2C \,/\, C_2C_1$

$C_1 \to a$

$C_2 \to \varepsilon$

$A \to C_1B \,/\, C_2E$

$B \to C_1A$

$C \to C_3CD$

$C_3 \to c$

$C \to XD$
$X \to C_3C$
$D \to C_1C_4C_5$
$C_4 \to b$
$C_5 \to d$
$D \to YC_5$
$Y \to C_1C_4$

c) Definition of non-generating and non-reachable symbols with example.

$S \to AB \mid a$
$A \to aA$
$B \to b$

A variable is non-generating if it does not derive any string of terminals. In the above example A is non-generating symbol.
A variable is non-reachable if the start symbol does not derive a string which contains that variable.
If $X \to w$, A symbol y is unreachable if w string does not contain y. In the above example B is unreachable after deleting non-generating symbol A from production.

5. Prove that Context Free Languages are not closed under intersection. **[WBUT 2014, 2017]**

Answer:

Consider the two languages

$$L_1 = \{a^n b^n c^m : n \geq 0\ m \geq 0\}$$

and $L_2 = \{a^n b^m c^m : n \geq 0\ m \geq 0\}$.

There are several ways one can show that L1 and L2 are context-free. For instance, a grammar for L1 is

S → S1 S2
S2 → aS1 b | λ
S2 → cS1 | λ

Alternatively, we note that L1 is the concatenation of two context – free languages, so it is context-free. But

L1 ∩ L2 = {an bn cn : n ≥ 0 },

which is not context-free . Thus, the family of context-free languages is not closed under intersection.

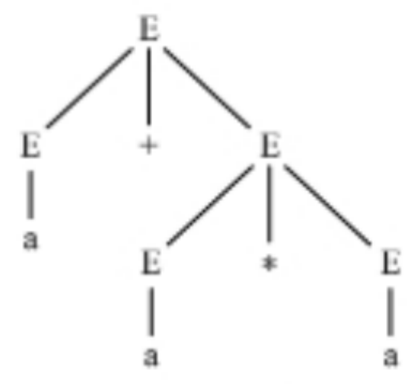
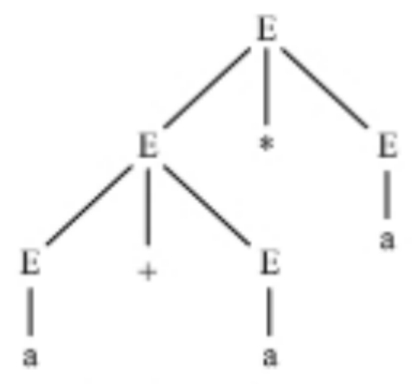
6. a) $E->E+E|E*E|a$. **Prove that the CFG with this production rule is ambiguous. Remove the ambiguity from this grammar.** **[WBUT 2018]**

Answer:

1st Part:

$E \to E + E \mid E * E \mid a$

$\therefore$ Say, string is: $a + a * a$

Derivation Tree 1	Derivation Tree 2
	

$\therefore$ So, more than one derivation tree for the string generated from the grammar.

$\therefore$ The given grammar is ambiguous.**(Ans.)**

2nd Part:

Removing ambiguity:

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to E \mid a$$

b) $S->AB;\ A->a;\ B->C/b, C->D;\ D->E, E->a$

Remove the unit production. **[WBUT 2018]**

Answer:

Refer Question No. 2(b) 1st Part of Long Answer Type Questions.

7. a) Convert the following Context-free grammar into an equivalent grammar in CNF: **[WBUT 2019]**

$$S \to 1A \mid 0B,\ A \to 1AA \mid 0S \mid 0,\ B \to 0BB \mid 1S \mid 1.$$

Answer:

Given that —

$$S \to 1A \mid 0B$$
$$A \to 1AA \mid 0S \mid 0$$
$$B \to 0BB \mid 1S \mid 1$$

Step 1:

The given grammar is already completely reduced.

Step 2:

The productions already in Chomsky normal form are —

$A \to 0$ .... (1)

$$B \to 1 \quad \ldots. (2)$$

These productions not in Chomsky normal form are —

$$S \to 1A \mid 0B \quad \ldots. (3)$$

$$A \to 1AA \mid 0S \quad \ldots. (4)$$

$$B \to 0BB \mid 1S \quad \ldots. (5)$$

We will convert these productions in Chomsky normal form.

Step 3:

Replace the terminal symbols 0 and 1 by new variables C and D.

This is done by introducing the following two new productions in the grammar —

$$C \to 0 \quad \ldots. (6)$$

$$D \to 1 \quad \ldots. (7)$$

Now, the productions (3), (4) and (5) modifies to —

$$S \to DA \mid CB \quad \ldots. (8)$$

$$A \to DAA \mid CS \quad \ldots. (9)$$

$$B \to CBB \mid DS \quad \ldots. (10)$$

Step 4:

Out of (8), (9) and (10) the productions already in Chomsky normal form are —

$$S \to DA \mid CB \quad \ldots. (11)$$

$$A \to CS \quad \ldots. (12)$$

$$B \to DS \quad \ldots. (13)$$

These productions will remain as they are

The productions not in Chomsky normal form are —

$$A \to DAA \quad \ldots. (14)$$

$$B \to CBB \quad \ldots. (15)$$

We will convert these productions in Chomsky normal form.

Step 5:

Replace AA and BB by new variables E and F respectively.

This is done by introducing the following two new productions in the grammar

$$E \to AA \quad \ldots. (16)$$

$$F \to BB \quad \ldots. (17)$$

Now, the productions (14) and (15) modifies to —

$$A \to DE \quad \ldots. (18)$$

$$B \to CF \quad \ldots. (19)$$

Step 6:

From (1), (2), (6), (7), (11), (12), (13), (16), (17), (18) and (19) the resultant grammar is

$$S \to DA \mid CB$$

$$A \to CS \mid DE \mid 0$$

$$B \to DS \mid CF \mid 1$$

$$C \to 0$$

$$D \to 1$$

$$E \to AA$$

$$F \to BB$$

7. b) Let G be the grammar

$$S \to aB \mid bA,\ A \to a \mid aS \mid bAA,\ B \to b \mid bS \mid aBB\ .$$

For the string $aaabbabbab$, find

i) **leftmost derivation and rightmost derivation**
ii) **derivation tree**
iii) **is this grammar ambiguous?** **[WBUT 2019]**

Answer:

G be the grammar

$S \to aB \mid bA,\ A \to a \mid aS \mid bAA,\ B \to b \mid bS \mid aBB$, given string is $aaabbabbab$

Parse Tree:

- The process of deriving a string is called as derivation.
- The geometrical representation of a derivation is called as parse tree or derivation tree.

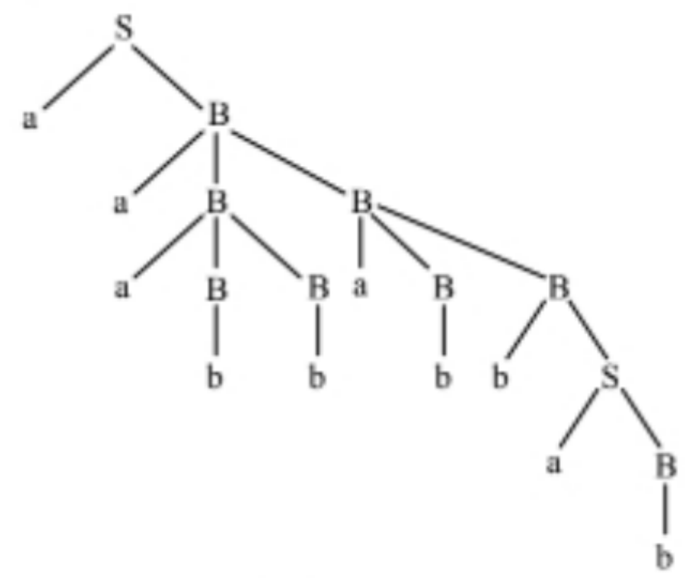
There are two types of derivation tree —

(i) Leftmost derivation, (ii) Rightmost derivation.

Now, leftmost derivation of the given grammar G is —

$$\begin{aligned}
S &\to a\underline{B} \\
&\to aa\underline{B}B \quad (B \to aBB) \\
&\to aaa\underline{B}BB \quad (B \to aBB) \\
&\to aaab\underline{B}B \quad (B \to b) \\
&\to aaabb\underline{B} \quad (B \to b) \\
&\to aaabba\underline{B}B \quad (B \to aBB) \\
&\to aaabbab\underline{B} \quad (B \to b) \\
&\to aaabbabb\underline{S} \quad (B \to bS) \\
&\to aaabbabba\underline{B} \quad (S \to aB) \\
&\to aaabbabbab \quad (B \to b)
\end{aligned}$$

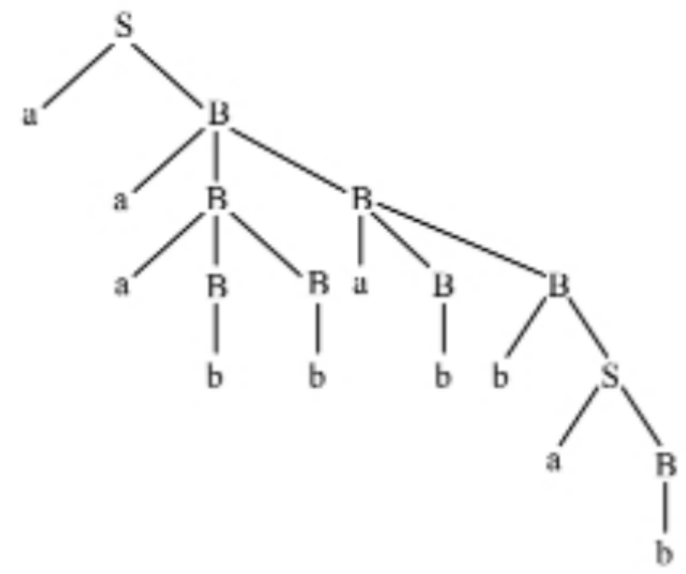
Leftmost Derivation Tree:



and Rightmost derivation of the given Grammar G is —

$$
\begin{aligned}
S &\to a\underline{B} \\
&\to aaB\underline{B} \quad (B \to aBB) \\
&\to aaBaB\underline{B} \quad (B \to aBB) \\
&\to aaBaBb\underline{S} \quad (B \to bS) \\
&\to aaBaBba\underline{B} \quad (S \to aB) \\
&\to aaBa\underline{B}bab \quad (B \to b) \\
&\to aa\underline{B}abbab \quad (B \to b) \\
&\to aaaB\underline{B}abbab \quad (B \to aBB) \\
&\to aaa\underline{B}babbab \quad (B \to b) \\
&\to aaabbabbab \quad (B \to b)
\end{aligned}
$$

Rightmost Derivation Tree:



Here, the given grammar was unambiguous.
That's why, leftmost derivation and rightmost derivation represents the same parse tree.

Leftmost Derivation Tree = Rightmost Derivation Tree

8. How to inherent ambiguity? **[MODEL QUESTION]**

Answer:

A CFL L is inherently ambiguous if all grammars for L are ambiguous.

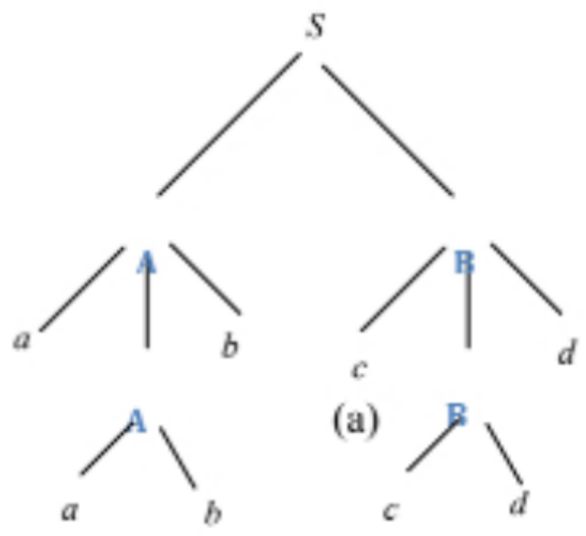
Example: Consider L =

$\{a^n b^n c^m d^m : n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n : n \geq 1, m \geq 1\}$.

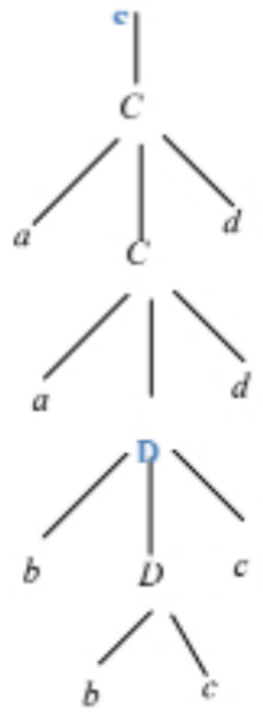
A grammar for L is

$$\begin{aligned} S &\rightarrow AB|C \\ A &\rightarrow aAb|ab \\ B &\rightarrow cBd|cd \\ C &\rightarrow aCd|aDd \\ D &\rightarrow bDc|bc \end{aligned}$$

Let's look at parsing the string aabbccdd.



(a)



(b)

From this we see that there are two leftmost derivations:

$$S \underset{lm}{\Rightarrow} AB \underset{lm}{\Rightarrow} aAbB \underset{lm}{\Rightarrow} aabbB \underset{lm}{\Rightarrow} aabbcBd \underset{lm}{\Rightarrow} aabbccdd$$

And

$$S \underset{lm}{\Rightarrow} C \underset{lm}{\Rightarrow} aCd \underset{lm}{\Rightarrow} aaDdd \underset{lm}{\Rightarrow} aabDcdd \underset{lm}{\Rightarrow} aabbccdd$$

It can be shown that *every* grammar for L behaves like the one above. The language L is inherently ambiguous.

9. Write a short note on Parse Tree. **[MODEL QUESTION]**

Answer:

- If $\omega \in L(G)$, for some CFG, then ω has a Parse tree, which tells us the (syntactic) structure of ω
- ω Could be a program, a SQL - query, an XML- document, etc.
- Parse trees are an alternative representation to derivations and recursive inferences.
- There can be several parse trees for the same string
- Ideally there should be only be only one parse tree (the "true" structure) for each string. i.e. the language should be unambiguous.
- Unfortunately, we cannot always remove the ambiguity.

Constructing Parse Trees:

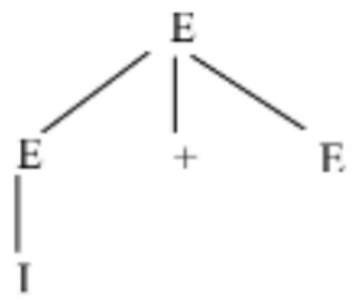
Let G = (V,T, P,S) be a CFG . A tree is a parse tree, for G if:

1. Each interior node is labeled by a variable in V.
2. Each leaf is labeled by a symbol in $V \cup T \cup \{\varepsilon\}$.
   Any $\in$ - labeled leaf is the only child of its parent.
3. If an interior node is labelled A, and its children (from left to right) labeled $X_1, X_2 \ldots, X_k$,
   then $A \to X_1 X_2 \ldots X_k \in P$.

Example: In the grammar

$$\begin{aligned} &1.\ E \to I \\ &2.\ E \to E+E \\ &3.\ E \to E*E \\ &4.\ E \to (E) \\ &\quad\vdots \end{aligned}$$

The following is a parse tree:



This parse trees shows the derivation $E \Rightarrow I + E$

The Yield of a Parse Tree

The yield of a parse tree is the string of leaves from left to right.

PUSHDOWN AUTOMATION

☞ Chapter at a Glance

PDA: A pushdown automata (PDA) is essentially an ε-NFA with a stack. On a transition the PDA

1. Consumes an input symbol.
2. Goes to a new state (or stays in the old).
3. Replaces the top of the stack by any string (does nothing, pops the stack, or pushes a string onto the stack).

Deterministic PDA: A PDA $P=(Q,\Sigma,\Gamma,\delta,q_0,Z_0,F)$ is *deterministic* if

1. $\delta(q,a,X)$ is always empty or a singleton.
2. If $\delta(q,a,X)$ is nonempty, then $\delta(q,\varepsilon,X)$ must be empty.

Multiple Choice Type Questions

1. The intersection of CFL and regular language **[WBUT 2006, 2012, 2019]**

a) need not be regular **b) need not be CF**
c) is always regular **d) none of these**

Answer: (d)

$L \cap R$ is a CFL. So it is correct, but (iv) should be the answer.

2. The class of context free language is not closed under **[WBUT 2008, 2009, 2013, 2019]**

a) concatenation **b) union**
c) intersection **d) repeated concatenation**

Answer: (c)

3. The grammar G=({S},{0,1}, P,S) where P=(S→0S1, S→0S, S→S1, S→0} is a **[WBUT 2008]**

a) recursively enumerable language **b) regular language**
c) context sensitive language **d) context free language**

Answer: (d)

4. Which of the following statements is wrong? **[WBUT 2010]**

a) A turning machine cannot solve halting problem
b) Set of recursively enumerable languages is closed under union
c) A finite state machine with 3 stacks is more powerful than infinite state machine with 2 stacks
d) Context sensitive grammar can be recognized by a linearly bounded memory machine

Answer: (c)

5. Context Free Grammar can be recognized by **[WBUT 2010]**

a) finite state automata
b) 2-way liner bounded automata
c) push-down automata
d) both (b) & (c)

Answer: (d)

6. A Push down automation is different from a finite automation because of **[WBUT 2011]**

a) a read head
b) a memory in the form of stack
c) a set of states
d) all of these

Answer: (b)

7. The intersection of CFL & RE is always **[WBUT 2014, 2015]**

a) CFL b) RE c) CSL d) CFL OR CSL

Answer: (a)

8. PDA is the machine format of **[WBUT 2018]**

a) Type 0 Language
b) Type 1 Language
c) Type 2 Language
d) Type 3 Language

Answer: (c)

9. The difference between finite automata and PDA _____. **[WBUT 2018]**

a) Reading Head
b) Input Tape
c) Finite Control
d) Stack

Answer: (d)

10. Useless symbols in CFG are **[WBUT 2018]**

a) Non-generating symbol and non-reachable symbols
b) Null alphabets and null string
c) Non-terminal symbols
d) All of the above

Answer: (b)

Short Answer Type Questions

1. Design a PDA which accepts the language.

L={*W* ε (a, b)* | *W* has equal number of a & b}. **[WBUT 2006, 2008, 2012, 2018]**

OR,

Construct a PDA accepting the set of all strings over $\{a, b\}$ with equal number of *a*'s and *b*'s. **[WBUT 2009, 2010, 2019]**

Answer:

Counting the number of a's and b's, which is easily done with a stack. Here we need not even worry about the order of the a's and b's. We can insert a counter symbol, say 0, into the stack whenever an a is read, then pop one counter symbol from the stack when a b is found. The only difficulty with this is that if there a prefix of w with more b's than a's, we will not find a 0 to use. But this is easy to fix; we can use a negative counter symbol,

say 1, for counting the b's that are to be matched against a's later. The complete solution is an nPDA

$M = (\{q_0, q_f\}, \{a,b\}, \{0,1,z\}, \delta, 0, z, \{q_f\})$ with δ given as

$\delta(q_0, \lambda, z) = \{(q_f, z)\}$, $\delta(q_0, a, z) = \{(q_0, 0z)\}$, $\delta(q_0, b, z) = \{(q_0, 1z)\}$,

$\delta(q_0, a, 0) = \{(q_0, 00)\}$, $\delta(q_0, b, 0) = \{(q_0, \lambda)\}$, $\delta(q_0, a, 1) = \{(q_0, \lambda)\}$

$\delta(q_0, b, 1) = \{(q_0, 11)\}$

In processing the string *baab*, the nPDA makes the moves

$(q_0, baab, z) \vdash (q_0, aab, 1z) \vdash (q_0, ab, z) \vdash (q_0, b, 0z) \vdash (q_0, \lambda, z) \vdash (q_f, \lambda, z)$

and hence the string is accepted.

2. Construct an NPDA that accepts the language generated by the productions $S \to aSa \mid bSb \mid c$. Show an Instantaneous Description of this string abcba for this problem. **[WBUT 2007, 2018]**

Answer:

$S \Rightarrow aSa$

$\Rightarrow abSba$

$\Rightarrow abcba$

3. What are the nonempty transitions in an NPDA? **[WBUT 2009, 2010, 2019]**

Answer:

Each move an NPDA reads a symbol from the input, changes the contents of the stack (that is, pop a symbol off and/or push several symbols into the stack), and changes the internal state of the npda.

Definition: An npda is M = def (Q; Σ; Γ; ±; q0; z; F), where

Q is a (finite, nonempty) set of states,

Σ is the (finite, nonempty) input vocabulary,

Γ is the (finite, nonempty) stack alphabet,

± : Q £ (Σ [f,g) £ Γ ! finite subsets of Q £ Γ

□

is the state transition function,

q0 2 Q is the initial state,

z 2 Γ is the initial stack symbol, and

F µ Q is the (finite, nonempty) set of accepting states.

An npda may include ¸-moves of the form ±(q; ¸; x).

Note that every moves pops off exactly one symbol from the stack.

4. Construct a Push Down Automata equivalent o the following Context Free Grammar $G = (Vn, Vt, P, S)$ **with** $Vn = \{S, E\}$, $Vt = \{0,1\}$ **and P is define transition follows:**

S-> 0S1/A

A-> 1A0/S/ε

Now check the string 001011 is acceptable by this Push Down Automata or not.

[WBUT 2015]

Answer:

1st Part:

The PDA is defined as follows:

A= $(\{q\}, \{0,1\}, \{S,A,0,1\}, \delta, q, S, \phi)$

δ is defined by the following rules:-

R1: $\delta(q, \Lambda, S) = \{(q,0S1)\}$

R2: $\delta(q, \Lambda, A) = \{(q,1S0),(q,S)\}$

R3: $\delta(q,0,0) = \{(q, \Lambda)\}$

R4: $\delta(q,1,1) = \{(q, \Lambda)\}$

2nd Part:

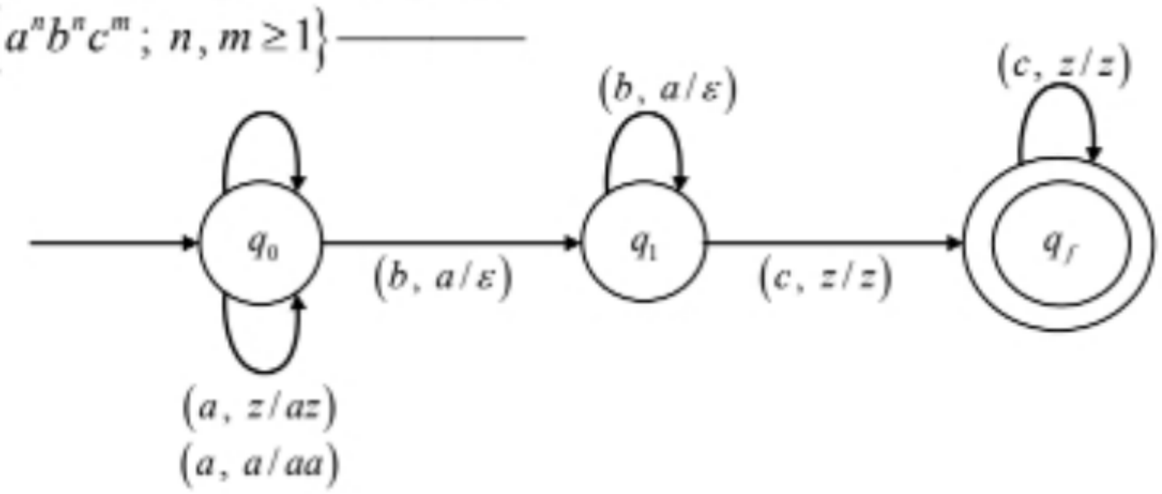
(q,001011,z) = (q, 01011,0z) = (q,1011,z)=(q,011,1z) => no rule is specified for the last transition . Hence the given string is not accepted by the PDA.

5. Construct a PDA to accept the language $L = \{a^n b^n c^m ;\ n, m \geq 1\}$ **by empty stack and by final state.** **[WBUT 2018]**

Answer:

Construct a PDA to accept the Language

$L = \{a^n b^n c^m ;\ n, m \geq 1\}$ ————



Transition Functions: **(Stack)**

$\left.\begin{aligned}\delta(q_0, a, z) &= (q_0, az)\\ \delta(q_0, a, a) &= (q_0, aa)\end{aligned}\right]$ push 'a' from the beginning

$\left.\begin{aligned}\delta(q_0, b, a) &= (q_1, \varepsilon)\\ \delta(q_1, b, a) &= (q_1, \varepsilon)\end{aligned}\right]$ pop whenever 'b' comes

$$\left.\begin{array}{l}\delta(q_1,c,z)=(q_f,z)\\ \underset{\text{(final state)}}{\delta(q_f,c,z)=(q_f,z)}\end{array}\right]$$ do, nothing for 'c', accepted by the empty stack

$\therefore$ Here, no. of a's are equal to
No. of b's $\Rightarrow$ so, push for 'a' and
pop for 'b' $\Rightarrow$ approach is used.

6. Construct a Deterministic pushdown automata that accepts the language $L=\{a^n cb^{2n}: n\geq 1\}$ over the alphabet $\Sigma=\{a,b,c\}$. **[MODEL QUESTION]**

Answer:
The NPDA

$$M=(\{q_0,q_1,q_2,q_f\},\{a,b,c\},(0,1),\delta,q_0,0,\{q_f\}) \text{ with}$$

$$\delta(q_0,a,0)=\{(q_1,110)\}$$
$$\delta(q_1,a,1)=\{(q_1,111)\}$$
$$\delta(q_1,c,1)=\{(q_2,111)\}$$
$$\delta(q_2,b,1)=\{(q_2,\lambda)\}$$
$$\delta(q_2,\lambda,0)=\{(q_f,\lambda)\}$$

Here only change the state stack element.

7. Construct an NPDA which generate the language **[MODEL QUESTION]**

$$L=\{a^n b^n : n\geq 0\}\cup\{a\}.$$

Answer:

$$Q=\{q_0,q_1,q_2,q_3\},$$
$$\Sigma=\{a,b\},$$
$$\Gamma=\{0,1\},$$
$$z=0,$$
$$F=\{q_3\}$$

and the transition function are

$$\delta(q_0,a,0)=\{(q_1,10),(q_3,\lambda)\},$$
$$\delta(q_0,\lambda,0)=\{(q_3,\lambda)\},$$
$$\delta(q_1,a,1)=\{(q_1,11)\},$$
$$\delta(q_1,b,1)=\{(q_2,\lambda)\},$$

$$\delta(q_2,b,1)=\{(q_2,\lambda)\},$$
$$\delta(q_2,\lambda,0)=\{(q_3,\lambda)\}$$

8. Construct an NPDA for the language **[MODEL QUESTION]**

$$L=\left\{w\in\{a,b\}^* : n_a(w)=n_b(w)\right\}.$$

Answer:

Counting the number of a's and b's, which is easily done with a stack. Here we need not even worry about the order of the a's and b's. We can insert a counter symbol, say 0, into the stack whenever an a is read, then pop one counter symbol from the stack when a b is found. The only difficulty with this is that if there a prefix of w with more b's *than* a's, we will not find a 0 to use. But this is easy to fix; we can use a negative counter symbol, say 1, for counting the b's that are to be matched against a's later. The complete solution is an nPDA

$M=(\{q_0,q_f\},\{a,b\},\{0,1,z\},\delta,0,z,\{q_f\})$ with δ given as

$$\delta(q_0,\lambda,z)=\{(q_f,z)\},$$
$$\delta(q_0,a,z)=\{(q_0,0z)\},$$
$$\delta(q_0,b,z)=\{(q_0,1z)\},$$
$$\delta(q_0,a,0)=\{(q_0,00)\},$$
$$\delta(q_0,b,0)=\{(q_0,\lambda)\},$$
$$\delta(q_0,a,1)=\{(q_0,\lambda)\}$$
$$\delta(q_0,b,1)=\{(q_0,11)\}$$

In processing the string *baab*, the nPDA makes the moves

$$(q_0,baab,z)\vdash(q_0,aab,1z)\vdash(q_0,ab,z)\vdash(q_0,b,0z)$$
$$\vdash(q_0,\lambda,z)\vdash(q_f,\lambda,z)$$

and hence the string is accepted.

9. State the applications of the pumping lemma. **[MODEL QUESTION]**

Answer:

The pumping lemma is extremely useful in proving that certain sets are not regular. To prove a language to be non-regular following steps are followed.

1) Select the language L you wish to prove non-regular.
2) The adversary picks n, the constant mentioned in the pumping lemma. You must be prepared in what follows for any finite integer n to be picked, but once the adversary has picked n, he may not change it.

3) Select a string z in L. Your choice may depend implicitly on the value of n chosen in (2).
4) The adversary breaks z into u, v and w, subject to the constraints that $|uv| \leq n$ and $|v| \geq 1$.
5) You achieve a contradiction to the pumping lemma by showing, for any u, v and w determined by the adversary that there exists an i for which $uv^i w$ is not in L. It may then be concluded that L is not regular. Your selection of i may depend on n, u, v and w.

Long Answer Type Questions

1. Define Pushdown Automata. [WBUT 2006, 2009, 2010, 2011, 2012, 2018, 2019]

Answer:

A **pushdown automaton** is a finite automaton that can make use of a stack containing data.

Pushdown automata differ from normal finite state machines in two ways:

1. They can use the top of the stack to decide which transition to take.
2. They can manipulate the stack as part of performing a transition.

Pushdown automata choose a transition by indexing a table by input signal, current state, and the top of the stack. Normal finite state machines just look by input signal and current state: they have no stack to work with. Pushdown automata add the stack as a parameter for choice. Given an input signal, current state, and a given symbol at the top of the stack, a transition path is chosen.

Pushdown automata can also manipulate the stack, as part of performing a transition. Normal finite state machines choose a new state, the result of following the transition. The manipulation can be to push a particular symbol to the top of the stack, or to pop off the top of the stack. The automaton can alternatively ignore the stack, and leave it as it is. The choice of manipulation (or no manipulation) is determined by the transition table.

2. Prove that CFLs are not closed under intersection and complement operation. [WBUT 2008, 2010, 2016, 2019]

Prove that the intersection of a context-free language and a regular language is a context-free language. [WBUT 2008, 2010]

OR,

Explain Ogden's Lemma for CFL. [WBUT 2013, 2016, 2019]

Answer:

Ogden's Lemma

In the theory of formal languages, **Ogden's lemma** provides an extension of flexibility over the pumping lemma for context-free languages.

Ogden's lemma states that if a language L is context-free, then there exists some number $p > 0$ (where p may or may not be a pumping length) such that for any string w in L, we can mark p or more of the positions in w as "distinguished"; then w can be written as

$w = uvxyz$

with strings u, v, x, y and z, such that v and y have at least one distinguished position between them, vxy has at most p distinguished positions, and

$uv^i xy^i z$ is in L for every $i \geq 0$.

Note that this is trivially true if the language is not infinite, since then p just needs to be longer than longest string in the language.

Ogden's lemma can be used to show that certain languages are not context-free, in cases where the pumping lemma for context-free languages is not sufficient. Observe that when every position is marked as distinguished, this lemma is equivalent to the pumping lemma for context-free languages.

Next Part:

Let L_1 be a context-free language and L_2 be a regular language. Then we have to prove that $L_1 \cap L_2$ is context-free.

Proof:

Let $M_1 = (Q, \Sigma, \Gamma, \delta_1, q_0, z, F_1)$ be an npda which accepts L_1 and $M_2 = (P, \Sigma, \delta_2, p_0, F_2)$ be a *dfa* that accepts L_2. We construct a pushdown automation $\bar{M} = (Q, \Sigma, \Gamma, \bar{\delta}, \bar{q}_0, \bar{z}, \bar{F})$ which simulates the parallel action of M_1 and M_2; whenever a symbol is read from the input string, $\bar{M}$ simultaneously executes the moves of M_1 and M_2. To this end we let

$$\bar{Q} = Q \times P \qquad q_0 = (q_0, p_0), \quad \bar{F} = F_1 \times F_2,$$

and define $\bar{\delta}$ such that $((q_k, p_t), x) \in \bar{\delta}((q_i, p_j), a, b)$,

iff $(q_k, x) \in \delta_1(q_i, a, b)$, and $\delta_2(p_j, a) = p_t$

In this, we also require that $a = \lambda$, then $p_j = p_t$. In the other words, the states of $\bar{M}$ are labeled with pairs (q_i, p_j), representing the respective states in which M_1 and M_2 can be after reading a certain input string. It is a straightforward induction argument to show that $((q_0, p_0), w, z) \vdash^*_{\bar{M}} ((q_r, p_s), x)$, with $q_r \in F_1$ and $p_s \in F_2$ if and only if

$(q_0, w, z) \vdash^*_{M_1} (q_r, p_s)$, and $\delta^*(p_0, w) = p_s$.

Therefore, a string is accepted by $\bar{M}$ if and only if it is accepted by M_1 and M_2, that is, if it is in $L(M_1) \cap L(M_2) = L_1 \cap L_2$.

3. Construct PDA for $L = \{ww^R : w \text{ belongs to } (0,1)^*\}$. **[WBUT 2011, 2012]**

OR,

Construct a PDA to accept L = {WW^R | W belongs to (a, b)* and W^R is reverse string of W} by empty stack and final state. **[WBUT 2013]**

Answer:

Hear the main concept is that the symbols are retrieved from a stack in the reverse order of their insertion. When reading the first part of the string we push consecutive symbols on the stack. For the second part, we compare the current input symbol with the top of the stack, continuing as long as the two match. Since symbols are retrieved from the stack in reverse of the order in which they were inserted, a complete match will be achieved if and only if the input is of the form ww^R.

An apparent difficulty with this suggestion is that we do not know the middle of the string, that is, where w ends and w^R starts. But the nondeterministic nature of the automaton helps us with this; the nPDA correctly guesses where the middle is and switches states at that point. A solution to the problem is given by

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, z, F\},$$
$$Q = \{q_0, q_1, q_2\},$$
$$\Sigma = \{a, b\},$$
$$\Gamma = \{a, b, z\},$$
$$F = \{q_2\}$$

The transition function can be visualized as having several parts: a set to push w on the stack.

$$\delta(q_0, a, a) = \{(q_0, aa)\},$$
$$\delta(q_0, b, a) = \{(q_0, ba)\},$$
$$\delta(q_0, a, b) = \{(q_0, ab)\},$$
$$\delta(q_0, b, b) = \{(q_0, bb)\},$$
$$\delta(q_0, a, z) = \{(q_0, az)\},$$
$$\delta(q_0, b, z) = \{(q_0, bz)\},$$

a set to guess the middle of the string, where the nPDA switches from state q_0 to q_1.

$$\delta(q_0, \lambda, a) = \{(q_1, a)\},$$
$$\delta(q_0, \lambda, b) = \{(q_1, b)\},$$

a set to match w^R against the contents of the stack,

$$\delta(q_1, a, a) = \{(q_1, \lambda)\},$$
$$\delta(q_1, b, b) = \{(q_1, \lambda)\},$$

and finally

$$\delta(q_1, \lambda, z) = \{(q_2, z)\},$$

to recognize a successful match.

The sequence of moves in accepting *abba* is

$$(q_0, abba, z) \vdash (q_0, baa, az) \vdash (q_0, ba, baz) \vdash (q_1, ba, baz) \vdash (q_1, a, az) \vdash (q_1, \lambda, z) \vdash (q_2, z)$$

The nondeterministic alternative for locating the middle of the string is taken at the third move. At that stage, the PDA has the instantaneous descriptions (q_0, ba, baz) and has two choices for its next move. One is to use $\delta(q_0, b, b) = \{(q_0, bb)\}$, and make the move

$$(q_0, baa, az) \vdash (q_0, a, bbaz)$$

The second is the one used above, namely $\delta(q_0, \lambda, b) = \{(q_1, b)\}$, only the latter leads to acceptance of the input.

4. Construct an equivalent PDA for the following CFG. **[WBUT 2013, 2016]**

$S \rightarrow aAB / bBA$

$A \rightarrow bS / a$

$B \rightarrow aS / b$

Show an ID for the string abbaaabbbab for the PDA generated with stack description.

Answer:

1st Part: Constructing the PDA

We note that the given grammar is already in Greibach Normal Form.

Hence, our PDA has only one state --- q (i.e., Q = {q}), which is also the start state and the final state ((i.e., F = Φ).

The input alphabet is clearly Σ = {a, b} and the stack alphabet consists of only the non-terminals, i.e. Γ = {S, A, B}, with S as the start symbol.

The required PDA is <Q, Σ, Γ, δ, q, S, Φ>, where δ is:

(q, a, S) = {(q, AB)}

(q, b, S) = {(q, BA)}

(q, a, A) = {(q, ε)}

(q, b, A) = {(q, S)}

(q, a, B) = {(q, S)}

(q, b, B) = {(q, ε)}

2nd Part:

Instantaneous Descriptions (IDs) form string abbaaabbbab.

(q, abbaaabbbab, S)

|- (q, bbaaabbbab, AB)

|- (q, baaabbbab, SB)

|- (q, aaabbbab, BAB)

|- (q, aabbbab, SAB)

|- (q, abbbab, ABAB)

|- (q, bbbab, BAB)

|- (q, bbab, AB)

|- (q, bab, SB)

|- (q, ab, BAB)

|- (q, b, SAB)

|- (q, ε, BAAB)

Since the ID is NOT (q, ε, ε) and there is no input to scan, the PDA rejects the string.

Note: The above ID corresponds to the below mentioned left-most derivation in the given grammar:

S è aAB è abSB è abbBAB è abbaSAB è abbaaABAB èabbaaaBAB è

abbaaabAB è abbaaabbSB è abbaaabbbBAB è abbaaabbbaSAB è
abbaaabbbabBAAB (No further derivation possible).

5. Define PDA by giving a block diagram. [WBUT 2014, 2017]
What is the difference between DPDA and NPDA? [WBUT 2014]
Design a Non Deterministic Pushdown Automata for accepting the string L = {(Set of all palindromes over a, b)} by Empty stack [WBUT 2014]

OR,

Design a non-Deterministic Pushdown Automata for accepting the string $L=\{WCW^R \mid W\varepsilon(a,b)^*\}$ and W^R is the reverse of W by Empty stack. [WBUT 2017]

Construct an equivalent PDA for the following Context Free Grammar.
S → aA
A → aABC/bB/a
C → c
Show an ID for the string aabbbc for the PDA generated. [WBUT 2014, 2017]

Answer:

1st Part:

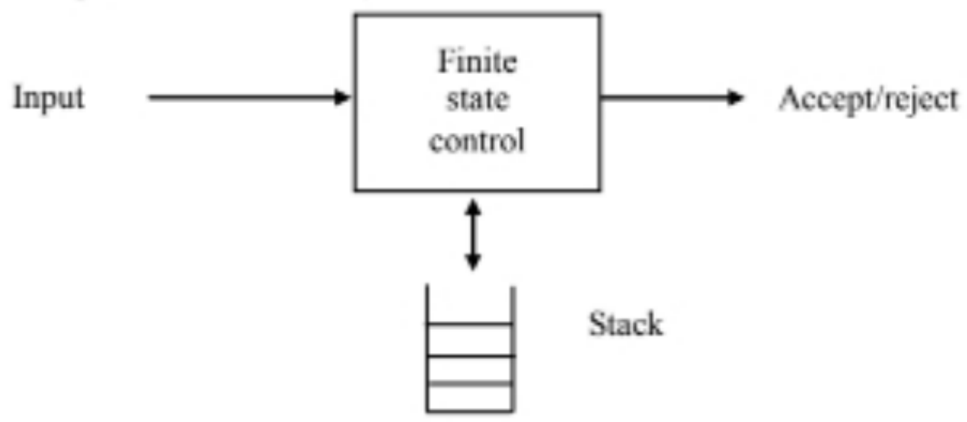
A pushdown automata (PDA) is a seven-tuple: $P=(Q,\Sigma,\Gamma,\delta,q_0,Z_0,F)$,

where

- Q is a finite set of states
- Σ is a finite input alphabet,
- Γ is a finite stack alphabet ,
- $\delta: Q\times\Sigma\cup\{\varepsilon\}\times\Gamma\to 2^{Q\times\Gamma^*}$ is the transition function,
- q_0 is the start state,
- $Z_0\in\Gamma$ is the start symbol for the stack, and
- $F\subseteq Q$ is the set of accepting states.

A PDA is essentially an ε-NFA with a stack.
On a transition the PDA

1. Consumes an input symbol.
2. Goes to a new state (or stays in the old).
3. Replaces the top of the stack by any string (does nothing, pops the stack, or pushes a string onto the stack).



2nd Part:
Difference between DPDA and NPDA

1. The transition function δ is atmost single-valued for DPDA, multi-valued for an NPDA. Formally: $\left|\delta(q_1, a, b)\right| = 0$ or 1, for every $q \varepsilon Q, a \in \Sigma \cup \{\varepsilon\}$ and $b \in F$.
2. Both NPDA and DPDA may have ε-transitions; but a DPDA may have a ε-transition only if no other transition is possible.

3rd Part:
Null string may belong to the Language set. The PDA will be designed in such a way that null string can be accepted by the PDA. The transitional functions will be

$\delta(q_0, a, z_0) \rightarrow (q_0, z_1 z_0)$,
$\delta(q_0, b, z_0) \rightarrow (q_0, z_2 z_0)$,
$\delta(q_0, a, z_1) \rightarrow (q_0, z_1 z_1), (q_1, \lambda)$,
$\delta(q_0, b, z_1) \rightarrow (q_0, z_1 z_1)$,
$\delta(q_0, a, z_2) \rightarrow (q_0, z_1 z_2)$,
$\delta(q_0, b, z_2) \rightarrow (q_0, z_2 z_2), (q_1, \lambda)$,
$\delta(q_1, a, z_1) \rightarrow (q_1, \lambda)$,
$\delta(q_1, b, z_2) \rightarrow (q_1, \lambda)$,
$\delta(q_0, \lambda, z_0) \rightarrow (q_1, \lambda)$ // this function is for accepting null string,
$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda)$ accepted by empty stack,
$\delta(q_1, \lambda, z_0) \rightarrow (q_f, z_0)$ accepted by final state.

4th Part: Question is wrong.

6. Explain how a string be accepted by a PDA? **[WBUT 2017]**

Answer:
There are two different ways to define PDA acceptability.

Final State Acceptability

In final state acceptability, a PDA accepts a string when, after reading the entire string, the PDA is in a final state. From the starting state, we can make moves that end up in a final state with any stack values. The stack values are irrelevant as long as we end up in a final state.

For a PDA (Q, ∑, S, δ, q_0, I, F), the language accepted by the set of final states F is −
L(PDA) = {w | (q_0, w, I) ⊢* (q, ε, x), q ∈ F}
for any input stack string **x**.

POPULAR PUBLICATIONS

Empty Stack Acceptability

Here a PDA accepts a string when, after reading the entire string, the PDA has emptied its stack.

For a PDA $(Q, \sum, S, \delta, q_0, I, F)$, the language accepted by the empty stack is −

$L(PDA) = \{w \mid (q_0, w, I) \vdash^* (q, \varepsilon, \varepsilon), q \in Q\}$

CONTEXT SENSITIVE LANGUAGES

☞ Chapter at a Glance

Context Sensitive Grammar: A **context-sensitive grammar** is a formal grammar $G = (N, \Sigma, P, S)$ such that all rules in P are of the form $\alpha A \beta \rightarrow \alpha \gamma \beta$ with A in N (i.e., A is single non-terminal) and α and β in $(N \cup \Sigma)^*$ (i.e., α and β strings of non-terminals and terminals) and γ in $(N \cup \Sigma)^+$ (i.e., γ a nonempty string of non-terminals and terminals), plus that a rule of the form $S \rightarrow \varepsilon$ with ε the empty string, is allowed if S does not appear on the right side of any rule. The name *context-sensitive* is explained by the α and β that form the context of A and determine whether A can be replaced with γ or not. This is different from a context-free grammar where the context of a non-terminal is not taken into consideration. A formal language that can be described by a context-sensitive grammar is called a ***context-sensitive* language**. That is one of the four types of grammars in the Chomsky hierarchy.

Properties of Context Sensitive Languages:

- The union, intersection, and concatenation of two context-sensitive languages is context-sensitive.
- The complement of a context-sensitive language is context-sensitive.
- Every context-free language is context-sensitive.

Multiple Choice Type Questions

1. The set A = $\{a^n b^n c^n \mid n = 1,2,3,....\}$ is an example of grammar that is **[MODEL QUESTION]**

a) Regular
b) Context free
c) Context sensitive
d) None of these

Answer: (c)

2. $L = \{a^n b^n c^n, \text{ where } n \in 1\}$ **is** **[MODEL QUESTION]**

a) regular
b) context free but not regular
c) context sensitive but not context free
d) none of these

Answer: (c)

3. The accepting automata for the context sensitive language is **[MODEL QUESTION]**

a) linear bounded automata
b) finite automata
c) push-down automata
d) all of these

Answer: (a)

4. The language L= (0n 1n 2n where n> 0) is a **[MODEL QUESTION]**

a) Regular language
b) Context-sensitive language
c) Context free language
d) Recursive enumerable language

Answer: (b)

5. The following grammar is **[MODEL QUESTION]**

$S \rightarrow a\alpha\, b \mid b\alpha\, c \mid \alpha\, B$
$S \rightarrow \alpha\, S \mid b$
$S \rightarrow \alpha\, bb \mid \alpha\, b$
$b\alpha \rightarrow bdb \mid b$

a) Regular b) Context-free c) Context sensitive d) LR (k)

Answer: (c)

6. The vernacular language English, if considered a formal language is a **[MODEL QUESTION]**

a) regular language
b) context – free language
c) context – sensitive language
d) none of these

Answer: (c)

Short Answer Type Questions

1. Show that every context sensitive language is recursive. **[MODEL QUESTION]**

Answer:

Suppose L is generated by the CSG G. The modified machine M′ still uses the markers <and>, and they remain in place; however, M′ is no longer prohibited from moving its tape head to the eight of >. The LBA M simulates a derivation in G, using the second "track" of the tape between the two markers. At the end of each iteration of this simulation, M′ performs two additional steps. First, M′ copies the current string of the derivation onto the empty portion of the tape to the right. In effect, this results in M′ keeping a history of the derivation, the string on the tape to the right of the > marker looks like this:

$$S\Delta\alpha_1\,\Delta\alpha_2\,\Delta \ldots \Delta\alpha_n$$

Where α_i is the current string after the ith iterations of the simulation. Second M′ determines whether α_n, the most recent string, is a duplicate of any of the previous α_is and crashes if it is.

With this modification, it is no longer possible for M′ to loop forever. Eventually, one of three things happens. Either the current string in the simulated derivation matches the original input x, in which case M′ halts; or an iteration of the loop results in a string longer than x, in which case M′ crashes; or one of strings obtained shows up for the second time (because all these strings are length |x| or shorter), in which case M′ crashes. The strings x for which some sequence of moves of M′ result in the first outcome precisely those that can be generated by G. We conclude that L is accepted by a NTM for which every sequence of moves ends in a halt or a crash.

2. Write down the properties of Context sensitive Language. Describe Context sensitive grammar. **[MODEL QUESTION]**

Answer:

1st Part:

- The union, intersection, and concatenation of two context-sensitive languages is context-sensitive.
- The complement of a context-sensitive language is context-sensitive.
- Every context-free language is context-sensitive.

2nd Part:

A context-sensitive grammar is a formal grammar $G = (N, \Sigma, P, S)$ such that all rules in P are of the form

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

with A in N (i.e., A is single non terminal) and α and β in $(N \cup \Sigma)^*$ (i.e., α and β strings of non terminals and terminals) and γ in $(N \cup \Sigma)^+$ (i.e., γ a nonempty string of non terminals and terminals), plus that a rule of the form

$S \rightarrow \varepsilon$

with ε the empty string, is allowed if S does not appear on the right side of any rule.

The name *context-sensitive* is explained by the α and β that form the context of A and determine whether A can be replaced with γ or not. This is different from a context-free grammar where the context of a non terminal is not taken into consideration. A formal language that can be described by a context-sensitive grammar is called a *context-sensitive* language.

That is one of the four types of grammars in the Chomsky hierarchy. Of the four, this is the least often used, in both theory and practice.

Computationally the context-sensitive languages are equivalent with linear bounded non-deterministic Turing machines. That is a non-deterministic Turing machine with a tape of only kn cells, where n is the size of the input and k is a constant associated with the machine. This means that every formal language that can be decided by such a machine is a context-sensitive language, and every context-sensitive language can be decided by such a machine.

3. Construct a context sensitive but not context-free grammar. **[MODEL QUESTION]**

Answer:

Let G = ({A, B, C, K}, {a, b} S, P)

Where the symbols have their usual meanings.

Here P consists of the following production rules:

$S \rightarrow AB|BA$

$AB \rightarrow a\,C$

$BA \rightarrow bK$

$C \rightarrow bK$

$C \rightarrow \wedge$

$K \rightarrow b$

Here, we have more than one non terminal at the L.H.S of the production rules.
Hence $\alpha \rightarrow \beta$
Where $\alpha = \beta = (V \cup T)^*$
Here $|\alpha| = 1$; this condition is not being satisfied.
Hence G is certainly context sensitive.

Long Answer Type Questions

1. Prove that,
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear bounded automation accepting L. **[MODEL QUESTION]**

Answer:

Suppose that G = (V, Σ, S, P) is a CSG generating L. We let the tape alphabet of our machine M contain pairs (a, b), where a, b $\in \Sigma \cup V \cup \{\Delta\}$, in addition to element of Σ other symbols may also be needed.

The first action taken by M is convert the tape configuration.

$\langle x_1\ x_2 \ldots x_n \rangle$

to

$\langle (x_1, \Lambda)\ (x_2, \Lambda) \ldots (x_n, \Lambda) \rangle$

Next, M places S in the second track of square 1 and starts a loop exactly as before, except that the machine crashes if the string produced in the second track during any iteration has length greater than n. As before M may exit the loop at any time. When it does, it crashes if the second track does not match the input in the first track. Because G is context-sensitive, a string appearing in the derivation of $x \in L$ cannot be longer than x, so that if the LBA begins with input x there is a sequence of moves it can execute that causes it to halt. Conversely, if $x \notin L$, then M either crashes or loops forever, because no simulated derivation is able to produce a string x.

TURING MACHINE & UNDECIDABILITY

☞ Chapter at a Glance

Turing Machine: A Turing Machine is a 7-tuple $M = \langle Q, \Gamma, b, \sum, \delta, q0, F \rangle$ where:

- Q is a *finite* set of states
- Γ is a finite set of the *tape* alphabet/symbols
- $b \in \Gamma$ is the *blank symbol*, the only symbol that is allowed to occur on the tape infinitely often at any step during the computation
- $\sum \subseteq \Gamma \setminus \{b\}$ is the set of *input symbols*
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is a partial function called the *transition function*, where *L* is head move left and *R* is head move right
- $q_0 \in Q$ is the *initial state*
- $F \subseteq Q$ is the set of final or *halting* states, accepting or rejecting.

Turing machine as Decider:

We have seen that the TM-s M_2, M_3 and M_4 that were used for computing integer functions did not have any FAIL-ure state. Also, it is clear that whatever be the inputs given (subject to conforming to our unary notation of numbers), the TM eventually comes to a SUCCess halt.

Functions like *NEXT*, *PREV*, *ADD*, etc., i.e., for which such TM-s can be constructed, are called **Primitive Recursive Functions.**

Any function that can be constructed as a finite number of compositions of other primitive recursive functions are themselves primitive recursive functions. For example, we can define the multiplication operation *PROD*(*m*, *n*) as: $PROD(m,n) = ADD(m,m),(n-1)\text{times}$

Turing machine as Enumerator:

The language of a Turing machine (when it is an enumerator) is the set of strings that the tm generates as output. For tm m, we often use the notation l(m) to denote the language enumerated by m.

A Language *L* is **Turing Recognizable** if there exists some TM which when given any string from *L* as input on its tape, will come to a success halt in finite time. If however, the input to the TM is NOT from *L*, the TM is not guaranteed to come to a failure halt in finite time.

Universal Turing Machine: The simulation by Ma proceeds as per the "program" described below:

1. Store s and p in the working memory

2. If s happens to be a SUCCess halt state of Mb, halt with success; else if s happens to be a FAILure halt state of Mb, halt with failure.
3. Note the current input of Mb (i.e., the symbol at the p-th position of Mb 's tape) and store it in the finite control (see Sec-14.5). Let this current input be x.
4. Search for the tuple (s, x, t, ω, d), i.e., whose si value is s and the a value is x.

5. Store t value, w and d values of the tuple in finite control
6. Proceed to the p-th position on the tape of Mb and write ω. Make $s \leftarrow t$. Make $p \leftarrow p+1$ if d is "right" else make $p \leftarrow p-1$
7. Goto Step-1

Looking at or description of Ma, we find that no matter what the description of Mb is, it can be simulated. In Formal Language parlance, a TM like Ma is known as a **Universal Turing Machine.**

Multiple Choice Type Questions

1. A shift register is **[WBUT 2013]**
a) Mealy M/C **b) Turing M/C** **c) Moore M/C** **d) All of these**
Answer: (d)

2. Difference between Turing Machine & Two way FA is in [WBUT 2014, 2015, 2017]
a) Input Tape **b) Read write head** **c) Finite Control** **d) All of these**
Answer: (d)

3. Which of the following statements is false? [WBUT 2014, 2015, 2017]
a) The halting problem of Turing machine is undecidable
b) Determining whether a context free grammar is ambiguous is undecidable
c) Given two arbitrary context free grammars G_1 and G_2. It is undecidable whether $L(G_1) = L(G_2)$
d) Given two regular grammars G_1 and G_2. It is undecidable whether $L(G_1) = L(G_2)$
Answer: (d)

4. Which is not a part of the mechanical diagram of 'Turing Machine'? [WBUT 2018]
a) Input Tape b) Read-write head
c) Finite control d) Stack
Answer: (d)

5. Which of the string is accepted by Turing Machine? [WBUT 2018]
a) $L = a^n c^m b^n$, where $m, n > 0$ b) $L = a^n b^n c^i$, $n, i > 0$
c) $L = a^n b^n c^n$, where $n > 0$ d) All of these
Answer: (d)

6. A PDM behaves like a TM when the number of auxiliary memory it has, is **[MODEL QUESTION]**

a) 1 or more
b) 0
c) 2 or more
d) None of these

Answer: (c)

Short Answer Type Questions

1. What do you mean by Halting problem of a Turing machine? [WBUT 2008, 2013]
OR,
Why a Turing machine is called linear bounded Automata? [WBUT 2008]
OR,
What is halting problem in Turing machine? [WBUT 2015, 2016, 2019]

Answer:

1st Part:

A Turing Machine is a mathematical object to define "Computing". It has a "doubly-infinite" tape for input and a state machine that governs its running. A Tuning Machine can behave like an algorithm, especially for problems with "yes/no" answer. In such a case, the machine starts with the description of a problem on the input tape and comes to a "Yes" halt or a "no" halt depending upon the result. However, it may happen that the machine never comes to any of he halts and go on running.

A related concept of "Universal Turing Machine" is a Turing machine that can take two things as input –

a) a coded description of the state machine of another Turing machine 'M' and
b) the input given to M. The Universal Turing Machine then proceeds to 'simulate' M.

The halting problem of Turing Machine is the question whether there can be a Universal Turing Machine that is given to simulate **ANY** machine 'M' with **ANY** input 'ω', with answer Yes / No depending upon whether M halts (i.e. M does not run forever) or does not halt, respectively, on input 'ω'.

Turing proved that the halting problem is unsolvable i.e. there can be no such machine as described.

2nd Part:

A linear bounded automata is anon deterministic turing machine satisfying the following two conditions.

1. Its input alphabet includes two special symbols ¢ and $, the left and right end markers respectively.
2. The LBA has no moves left from or right from $, nor may it print another symbol over ¢ or $.

The linear bounded automaton is simply a turning machine which instead of having potentially infinite tape on which to compute, is restricted to the portion of the tape containing the input x plus the two tape squares holding the end –markers.

An LBA will be denoted as
$M = (Q, \Sigma, \Gamma, \delta, q_0, ¢, \$, F)$ where $Q, \Sigma, \Gamma, \delta, q_0$ and F are as for a non-deterministic TM; ¢ and \$ are symbols in Σ, the left and right end-markers. L(M), the language accepted by M, is $\{w|w \text{ is in } (\Sigma-\{¢, \$)^* \text{ and } q_0 ¢ w \$ \overset{*}{\underset{M}{\mapsto}} \alpha q \beta \text{ for some q in F}\}$

2. Construct a Turing machine that accepts all strings over {0, 1} with an even number 0's and even number of 1's. **[WBUT 2011]**

Answer:

The requited Turing Machine is

$$M = \langle \{q_0, q_1, q_2, q_3, q_{SUCC}, q_{FAIL}\}, \Gamma, \Lambda, \{0,1\}, \delta, q_0, \{q_{SUCC}, q_{FAIL}\} \rangle$$

where δ is as given below:

$$\delta(q_0, 0) = (q_1, 0, R)$$
$$\delta(q_0, 1) = (q_3, 0, R)$$
$$\delta(q_0, \Lambda) = (qSUCC, \Lambda, L)$$
$$\delta(q_1, 0) = (q_0, 0, R)$$
$$\delta(q_1, 1) = (q_2, 0, R)$$
$$\delta(q_1, \Lambda) = (qFAIL, \Lambda, L)$$
$$\delta(q_2, 0) = (q_3, 0, R)$$
$$\delta(q_2, 1) = (q_2, 0, R)$$
$$\delta(q_2, \Delta) = (qFAIL, \Delta, L)$$
$$\delta(q3, 0) = (q_2, 0, R)$$
$$\delta(q_3, 1) = (q_0, 0, R)$$
$$\delta(q_3, \Delta) = (qFAIL, \Delta, R)$$

3. Design a Turing Machine that recognizes the language of all string of even length over the alphabet {a, b}. **[WBUT 2013]**

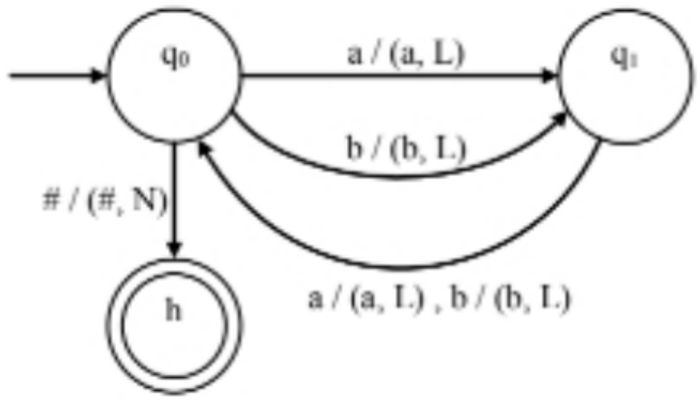
Answer:

Let Turing Machine $(T) = (Q, \Sigma, \Gamma, \delta, q_0, h)$

$$\Sigma = \{a, b\}$$
$$\Gamma = \{a, b, \#\}$$
$$Q = \{q_0, q_1, h\}$$

Here is h half state when machine halts after accepting the language, q_0 is initial state

	a	b	#
q_0	(q_1, a, L)	(q_1, b, L)	$(h, \#, N)$
q_1	(q_0, a, L)	(q_0, b, L)	Undefined move
h	Undefined move	Undefined move	Accept



4. State and State Transition of a Turing Machine **[MODEL QUESTION]**

Answer:

The control unit of a TM is a state-machine with a *finite* number of states.

In the beginning, the TM is in a state called the *initial state*. At this time, the characters of the *input string* (or simply, *input*) to this TM is written on the tape with a distinctive *blank* character marking the

end of the input. It is possible that the input string is "null", i.e., there is no or *empty* input. The head of the TM initially is on the first character of the input (or on the blank character if there is no input).

If the *current* state is an *accept state*, the TM *stops* or *halts* and we say that it has come to a **Success Halt**.

Similarly, if the *current* state is an *reject state*, the TM also *stops* or *halts* and we say that it has come to a **Failure Halt**.

5. Define the terms: Recursively Enumerable Set. **[MODEL QUESTION]**

Answer:

The class of ("decision")problems for which it is possible to formulate a Turing Machine that will *definitely* come to a halt (success or failure) in finite time, is called the **Recursive Set**. Eventually if the answer to the decision is 'yes', is called the **Recursively Enumerable Set**.

In the definition of recursively enumerable set, we have not ensured that if the answer is 'no', the Turing Machine will definitely halt.

6. Turing Decidable" is a subset of "Turing Recognizable".-Explain. **[MODEL QUESTION]**

Answer:

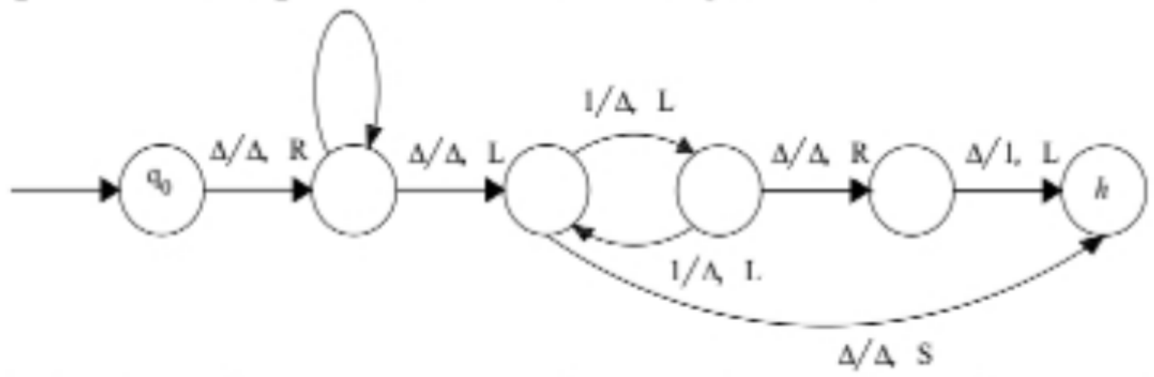
A Language L is **Turing Recognizable** if there exists some TM which when given any string from L as input on its tape, will come to a success halt in finite time. If however, the input to the TM is NOT from L, the TM is not guaranteed to come to a failure halt in finite time.

A Language L is Turing Decidable if there exists some TM which when given any string from L as input on its tape, will come to a success halt in finite time AND if the input to the TM is NOT from L, the TM will come to a failure halt in finite time.
Clearly "Turing Decidable" is a subset of "Turing Recognizable".

7. Design a Turing Machine to compute n mode 2. **[MODEL QUESTION]**

Answer:

The TM that performs computation of n mod 2 is given below:



The numeric function that assigns to each natural number n the remainder when n is divided by 2 can be computed by moving to the end of the input string, making a pass from right to left in which the 1's are counted and simultaneously erased, and either leaving a single 1 or leaving nothing.

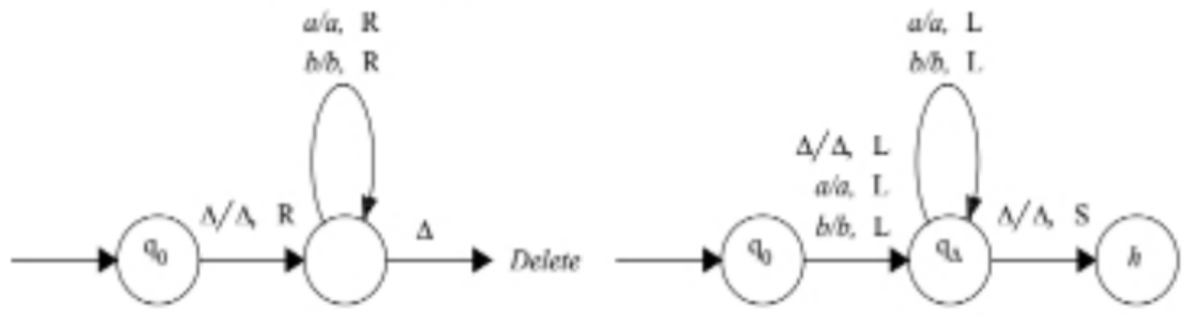
8. Design a Turing Machine for concatenating strings. **[MODEL QUESTION]**

Answer:

Consider the concatenation function

cat: $\Sigma^* \times \Sigma^* \rightarrow \Sigma^*$

A TM computing this function is obtained simply from the delete TM of Q.20 and is given in figure below for alphabet {a, b}.



Because natural numbers are represented by strings of 1's, and because in unary notation adding two natural numbers corresponds to concatenating the two strings, a simplified one-symbol version of this TM computes the addition function from $N \times N$ to N.

9. Proof that L_d is not recursively Enumerable. **[MODEL QUESTION]**

Answer:

Suppose L_d were L(M) for some TM M. Since L_d is a language over alphabet {0,1}, M would be in the list of Turing machines we have constructed, since it includes all TM's with input alphabet {0,1}. Thus there is atleast one code for M, say I; that is, $M = M_i$.
Now,ask if w_i is in L_d

1. If w_i is in L_d, then M_i accepts w_i. But then, by definition of L_d, w_i is not in L_d ,because L_d contains only those w_j such that M_j does not accept w_j.

10. Define PCP. **[MODEL QUESTION]**

Answer:

PCP stands for POST CORRESPONDENCE PROBLEM. The Post Correspondence Problem (PCP), introduced by Emil Post in 1946, is an undecidable decision problem. The PCP problem over an alphabet $\sum$ is stated as follows. Given the following two lists, **M** and **N** of non-empty strings over $\sum$ –

$M = (x_1, x_2, x_3, \ldots\ldots\ldots, x_n)$

$N = (y_1, y_2, y_3, \ldots\ldots\ldots, y_n)$

We can say that there is a Post Correspondence Solution, if for some $i_1, i_2, \ldots\ldots\ldots i_k$, where $1 \leq i_j \leq n$, the condition $x_{i1} \ldots\ldots x_{ik} = y_{i1} \ldots\ldots y_{ik}$ satisfies.

Fig: Reduction proving the undecidability of Post's Correspondence Problem

It consists of two lists of strings over some alphabet $\sum$; the two list must be of equal length. This instance of PCP has a solution, if there is a sequence of one or more integers that when interpreted as indexes for strings in the A and B lists, yield the same string. The Post's correspondence problem is: an instance of PCP, tell whether this instance has a solution.

Long Answer Type Questions

1. a) Design a TM that accepts $\{0^n 1^n | n \geq 1\}$ **[WBUT 2013, 2016]**

b) Design a TM which can multiply two positive integers. **[WBUT 2013]**

Answer:

a) Let us now follow the working of TM M_1 with input **000111**.

The initial configuration is **bS000111b**. This means that the TM is in state S and the head is on the first 0 of the input string. This corresponds to M_1 being as given in Fig - 1. The *current state* is mentioned in the box representing the finite control of the machine.

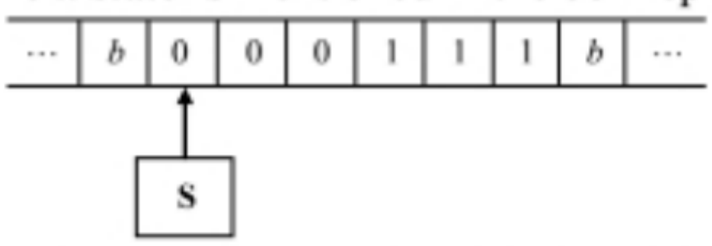


Fig: 1 Machine M_1 in Initial Configuration

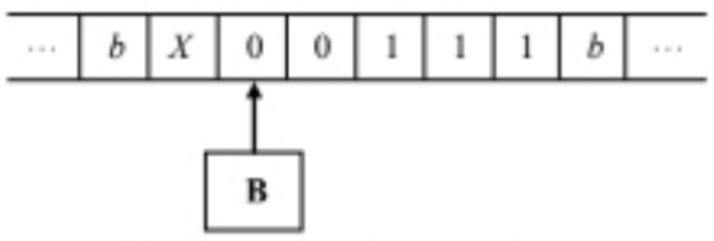


Fig: 2 M_1 After First Move

Since the input is 0, consulting the state table, we find that the move is (*B, X, R*), i.e., the next state is *B*, the 0 under the tape is overwritten by an *X* and the head moves *right* to place itself on the next (i.e., second) 0 of the input. This gives us the next configuration as

bXB00111b, as shown pictorially in Fig - 2.
Following up with the next moves, we find that the subsequent configurations are:
bX0B0111b followed by **bX00B111b**. The state of the machine in the last of the above two configurations is as seen in Fig - 3.

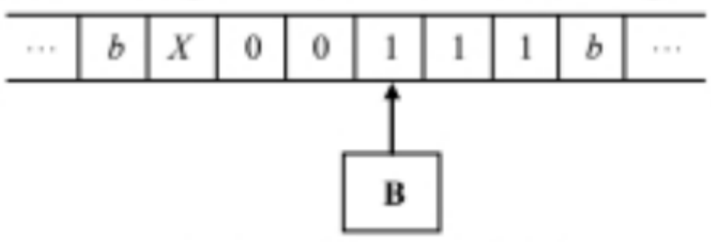


Fig: 3 M_1 When on First 1

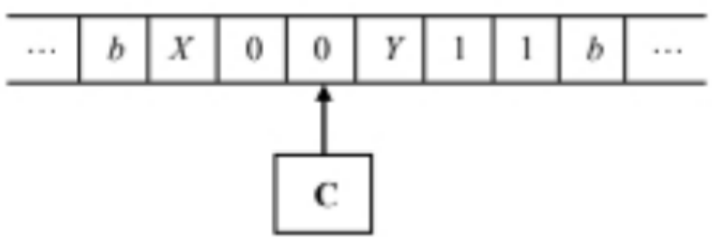


Fig: 4 M_1 After Marking First 1 to Y

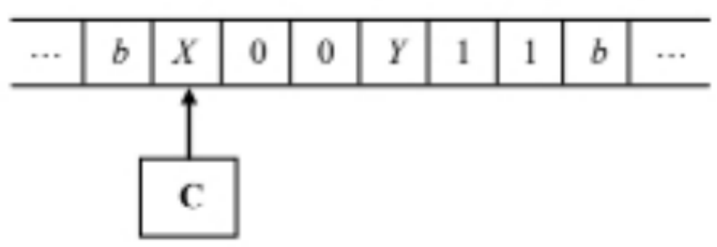


Fig: 5 M_1 After Matching One Pair of 0 and 1

At this point, we find the applicable move is (C, Y, L) which means that the next configuration is bX0C0Y11b. Observe in Fig - 4 that the machine has moved left after marking the first 1 to Y.

At this point, the next applicable move. i.e., $(C, 0, L)$, will effectively *skip* over the 0-s till the head is positioned on the first (up to now, the only) X encountered. The configurations leading to that
are **bXC00Y11b** followed by **bCX00Y11b**. The machine position now is as given in Fig - 5.

The next move is (S, X, R) and hence the next configuration is **bXS00Y11b**.
Let us summarize what the machine has done up to this point:

1. It has marked the first 0 of the input string to X
2. Then it has skipped the following 0-s while moving *right* till it encountered the first 1 which was marked to Y and the machine moved *left*
3. Now the machine skipped all 0-s while moving left till it got an X upon which it moved right and reached a condition similar to what was at the beginning of the run as shown in Fig - 6.

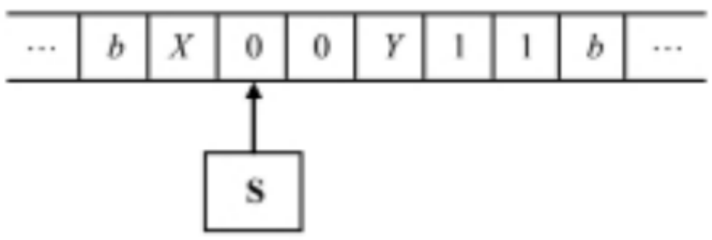


Fig: 6 M_1 Back in State S

Let us now augment the above summarization with the emphasized phrases:

1. It has marked the first 0 of the input string to *X*. However, if there is no 0 to mark but there is a Y below the head then it moves right over the *Y* -s till it reaches the end of input whence it halts with SUCCess
2. Then it has skipped the following 0-s or Y -s while moving right till it encountered the _rst 1 which was marked to Y and the machine moved left
3. Now the machine skipped all 0-s while moving left till it got an X upon which it moved right and reached a condition similar to what was at the beginning of the run. *However, if there is no 0 to skip, it halts with* ***SUCCess***

If you properly visualize the moves, you will see that our Turing Machine M_1:

- Checks out one 0 to X and one 1 to Y in each zigzag movement over the input.
- Declares SUCCess if and only if the number of 0-s marked as *X* is *exactly the same as* the number of 1-s marked as Y

Hence our TM recognizes only inputs that are of the form $0^n1^n, n \geq 1$.

b) Design a TM, which can multiply two positive integers

We can use subroutines for design of TM which can multiply two positive integers

$0^m\,1\,0^n$ [$m \times n$: operators are represented with 1] [If $m=3, n=2,\ m \times n = 6$]

So at the final state the TM will halt with 6 zeros and all surrounded by Blank B.

i) If First 0 is encountered by q_0 it is changed to Blank B

B $\dot{0}$ 0 1 0 0 1 B B B B

ii) Move right until first 1 is encountered, change the immediate next 0 to X

B 0 0 1 X 0 1 B B B B

iii) Move right and skip all symbol until first blank is encountered, then change the blank to 0 (as if 0 is copied here) – it is called subroutine call

B 0 0 1 X 0 1 0 B B B

iv) Move left until X is encountered, change the next 0 to X

B 0 0 1 X X 1 0 B B B

v) Move right until B is encountered and change B to 0

B 0 0 1 X X 1 0 0 B B

vi) Move left until X reached and change consecutive X to 0's

B 0 0 1 0 0 1 0 0 B B

vii) Move to left until B is reached, change the next 0 to B

B B 0 1 0 0 1 0 0 B B

viii) Move right until 1 reached change the next 0 to X, (subroutine)

B B 0 1 X 0 1 0 0 B B

ix) Move right until B reached, change that B to 0

B B 0 1 X 0 1 0 0 0 B B .....

x) Move left until X reached, change next 0 to X and repeat the procedure as above.

BB01XX1000BB
BB01XX10000BB....
BB010010000BB....
BBB10010000BBB....
BBB1X010000BB....
BBB1X0100000B....
BBB1XX100000BBB....
BBB1XX1000000BBB....
BBB10010000000BBBB
BBB1001000000B....

So, now no more 0's encountered, then change all the 1 & 0's on left to B.

$$\text{Halt} \rightarrow 0/B \qquad \left[\text{BBBBBBB}\,|\,000000\,|\,\text{B}\ldots\ldots\right].$$

2. Design a Turing Machine which performs addition of two integers. Write short note on Multi Tape and Multi Head Turing Machine. Prove that the problem "A string w halts on a Turing Machine M" is undecidable. [WBUT 2014]

Answer:

1st Part:

The turning machine will have 0^n10^n on its tape initially. Therefore it will start with the leftmost 0, so on scanning 0's moving right keeping them as if, till if get 1, if replace this 1 by 0 and then again so on. Scanning 0's and moving right keeping them as it is. When it gets a B (blank), it moves left and replaces the right most 0 by B and halfs thereby leaving finally $m+n$ 0's on the tape. Therefore, the moves of the Turing machine are

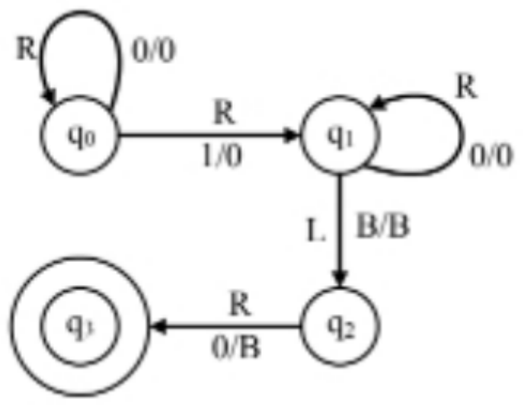
	0	1	B
q_0	$(d_0, 0, R)$	$(d_1, 0, R)$	—
q_1	$(d_1, 0, R)$	—	q_2, B, L
q_2	q_3, B, R	—	—
q_3	—	—	—

Therefore the Turing machine

$$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_3\})$$

where δ is given above.

The transition diagram corresponding to the above table is



2nd Part:

A Multi-tape Turing machine is like an ordinary Turing machine with several tapes. Each tape has its own head for reading and writing. Initially the input appears on tape 1, and the others start out blank.

A k-tape Turing machine can be described as a 6-

$$M = \langle Q, \Gamma, s, b, F, \delta \rangle$$

where

Q is a finite set of states

Γ is a finite set of the tape alphabet

$s \in Q$ is the initial state

$b \in \Gamma$ is the blank symbol

$F \subseteq Q$ is the set of final or accepting states

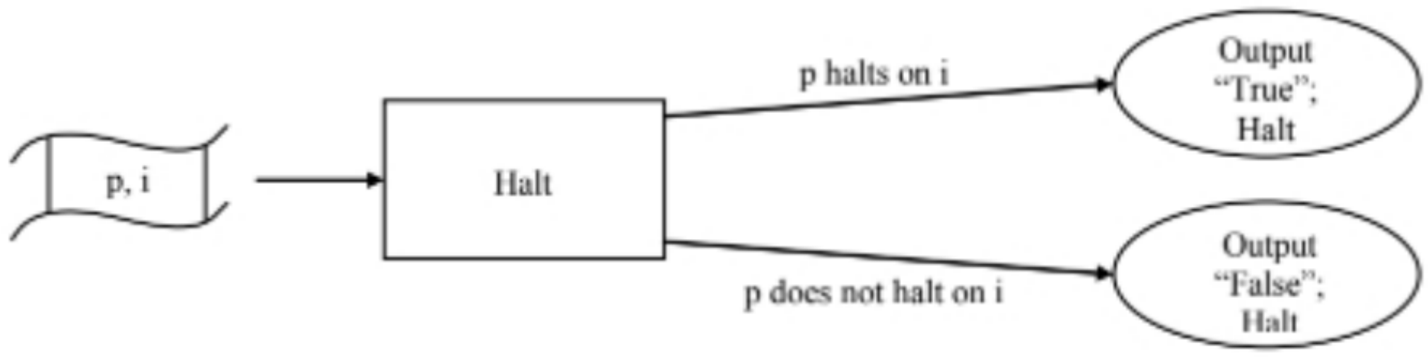
$\delta : Q \times \Gamma^k \to (\Gamma \times \{L, R, S\})^k$ is a partial function called the transition function, where k is the number of tapes, L is left shift, R is right shift and S is no shift.

A multihead Turing machine can be visualized as a Turing machine with a single tape and a single control unit but with multitape.

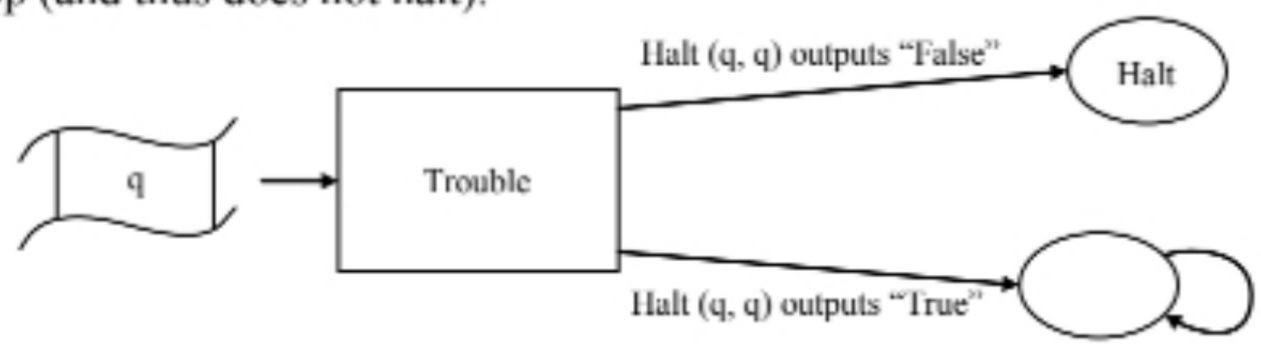
3rd Part:

Here is a sketch of how you can prove that the Halting problem is undecidable.

Assume that a TM capable of solving the halting problem exists. Call this TM halt. It takes, as input, the encoded representation of a TM and an input tape on which the encoded TM will run. Let's call this input pair (p, i): p for "program" (the encoded TM) and i for "input" (the input tape). If input TM p will halt on input i, then halt halts and outputs "true". If input TM p will not halt on i, then halt halts and outputs "false". So, we can view halt as a function that takes parameters p and i and produces the output "true" or "false" depending on whether or not TM p halts on input i.



Based on halt, we can trivially construct a new TM, which we can call trouble, which will work as follows. It first duplicates the entire input tape q, creating two copies of the original input tape. It then runs halt, using one copy of the original input q as parameter p and the other copy of the original input q as parameter i. If halt (q, q) outputs "false", then trouble TM halts. If halt (q, q) outputs "true", then trouble goes into an infinite loop (and thus does not halt).



Any TM can be encoded as an initial tape. So, let's assume that the encoding of trouble as a tape is called t.

Consider what will happen when trouble is executed with t as its input tape. Does trouble halt?

If trouble halts, that means that halt (t, t) answered "false". However, that means that trouble does not halt when given t as input.

If trouble does not halt, that means that halt (t, t) answered "true". However, that means that trouble does halt when given t as input.

In either case, halt gave the wrong answer. Therefore, given any TM that claims to solve the halting problem, it is possible to construct a program/input pair for which it will answer incorrectly. So, any claim that a particular TM solves the Halting problem can be proved false, meaning that the Halting Problem is undecidable.

3. a) Define Turing machine. **[WBUT 2015, 2019]**
b) Explain different types of Turing machine. **[WBUT 2015, 2019]**
c) Design a Turing machine that accepts the language of all string which contain "aba" as a substring. **[WBUT 2015, 2019]**

Answer:

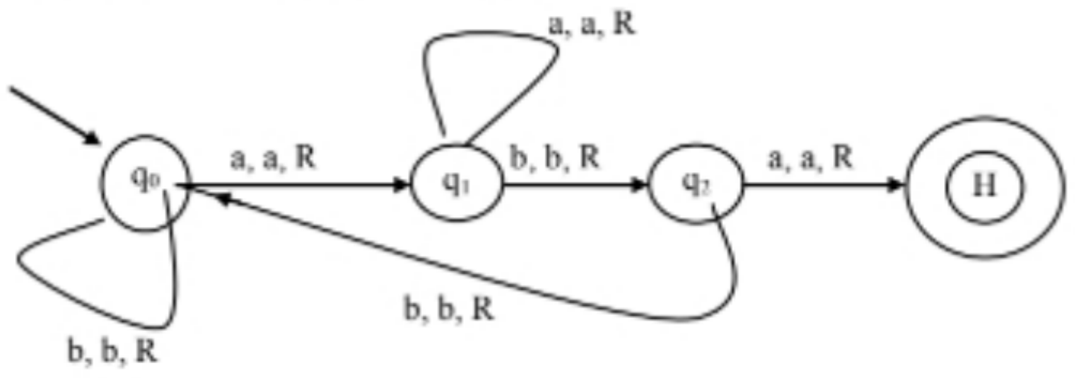
a) A Turing machine is an abstract concept used to describe a type of machine that, given an indefinite amount of space and time, can be adapted to calculate anything, such as the digits of π or even a whole universe.

b) There are many different types of Turing machines that are often used to describe certain kinds of execution. The two that are most often used are deterministic and non-deterministic Turing machines.
Automata are often used to represent Turing machines. This is a deterministic Turing machine which calculates the two's complement of some binary input.
A deterministic Turing machine is one that uses the concept of determinism to calculate a solution to a problem.

Determinism is the concept of being only in one state at a given time and determining what the next state will be from that state. In simpler terms, determinism would be being in state q_0, and only holding that state until moving onto the next state, q_1. In determinism we would be able to predict without any doubt that the head would move from state q_0 to state q_1. A Turing machine does not have to even halt, or stop execution, in order for it to be considered deterministic.

A non-deterministic Turing machine is one that uses the concept of non-determinism to calculate a solution to a problem.
A non-deterministic Turing Machine differs from a deterministic Turing Machine in the sense that a non-deterministic Turing Machine can have several possible states to which it can transition from any given state, q_i. One way to think of it would be to think that, given the possibility of choosing from several subsequent states, the non-deterministic Turing machine guesses the next iteration that will bring it to a 'yes' answer. Put a different way, the non-deterministic Turing machine branches out into holding many states at the same time in a sort of tree fashion until one of the many paths leads it to a 'yes' answer. In perspective a non-deterministic Turing machine may, for instance, be in state q_0 and then hold both state q_{11} as one of the branches and state q_{12} as another branch.

c) Strings in which aba is present as a substring.



4. Design a Turing Machine which accepts the language $L=\{a^n b^n, n \geq 1\}$. Write a short note on Multi-Tape and Multi Head Turing Machine. **[WBUT 2017]**

Answer:
1st Part:

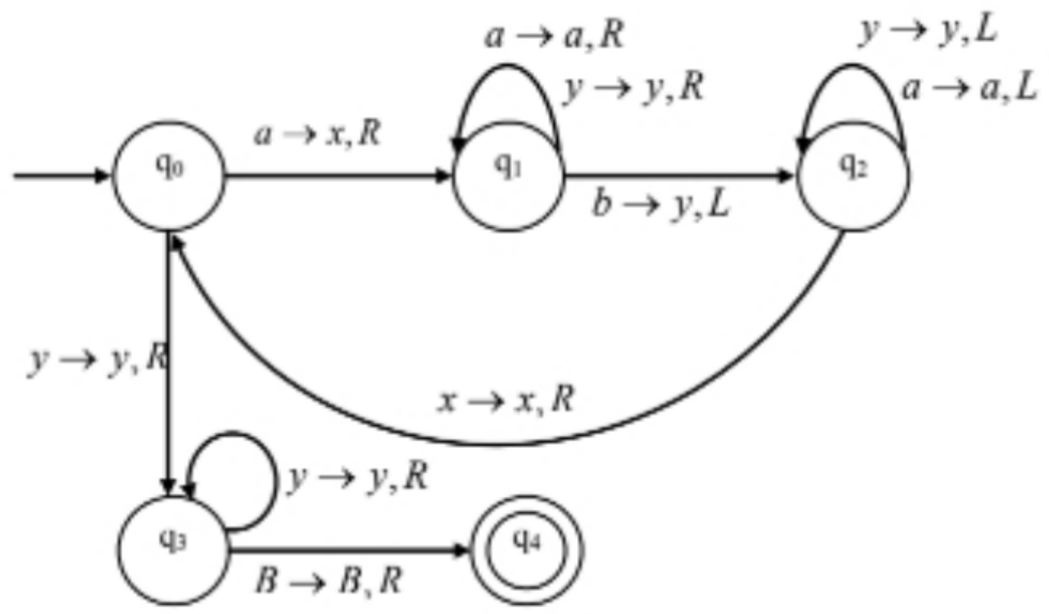


Fig: 1

Let us now follow the working of TM M_1 with input **aabb**.
The initial configuration is **BaabbBB**. This means that the TM is in state q_0 and the head is on the first **a** of the input string. This corresponds to M_1 being as given in Fig: 2(a). The *current state* is mentioned in the box representing the finite control of the machine.

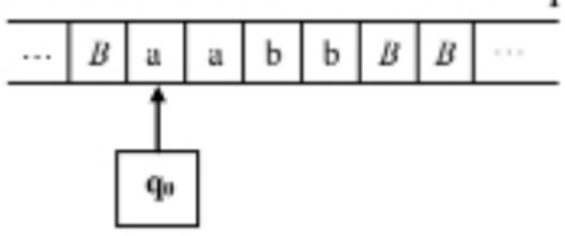


Fig: 2(a) Machine M_1 in Initial Configuration

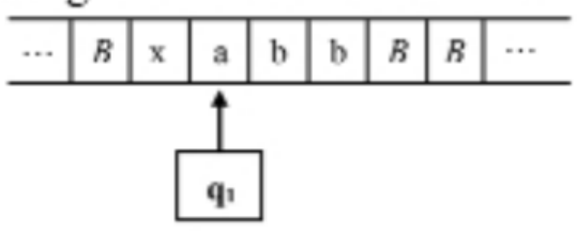


Fig: 2(b) M_1 After First Move

After the first move, the head q_0 reads a, removes it with x and moves to right, thus changing the state from q_0 to q_1. This gives us the next configuration as **BxabbBB**. Since there is another **a** in the second state, so we will overwrite the **a** with **a** and move to right ($a \to a, R$).

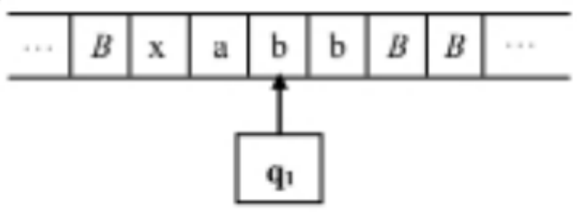


Fig: 2(c) M_1 After Second Move

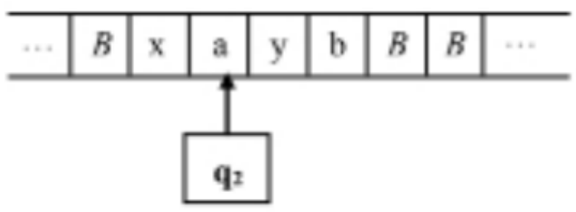


Fig: 2(d)

After the second move the configuration becomes **BxabbBB** with the head pointing to b. So it reads b, removes it with y and moves to left, thus changing the state from q_1 to q_2. Following the same procedure the turing machine is designed. The next moves are as follows:

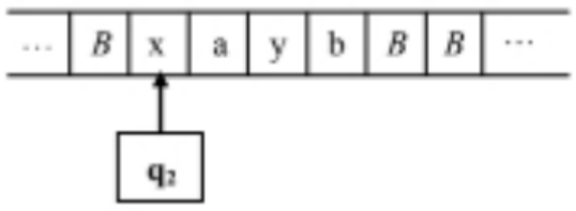


Fig: 2(e)

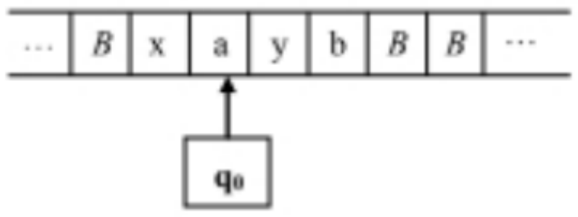


Fig: 2(f)

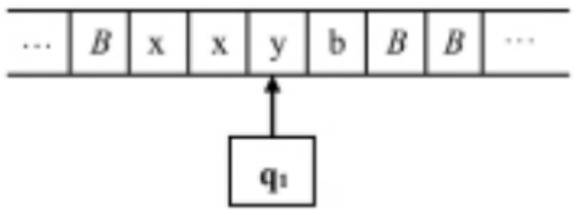


Fig: 2(g)

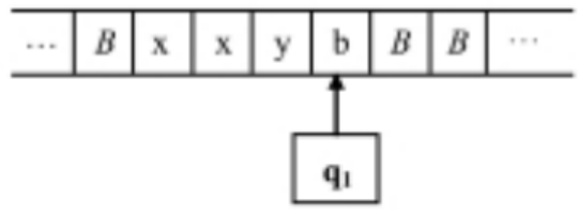


Fig: 2(h)

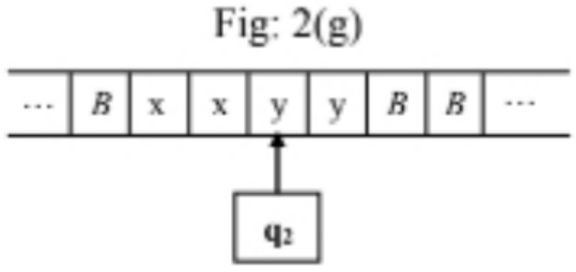


Fig: 2(i)

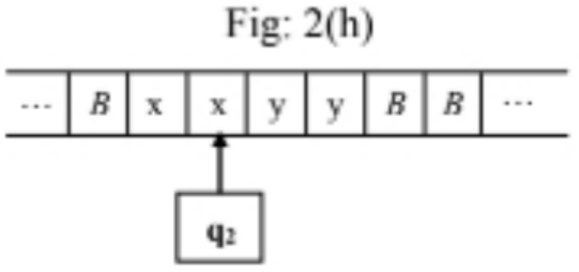


Fig: 2(j)

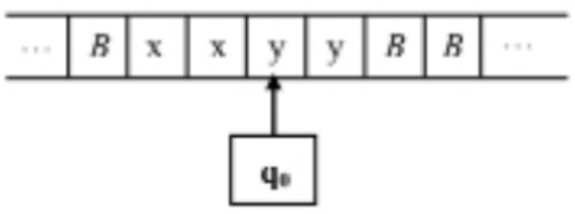


Fig: 2(k)

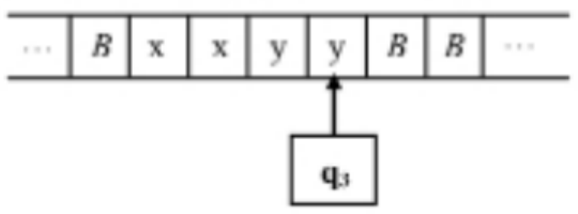


Fig: 2(l)

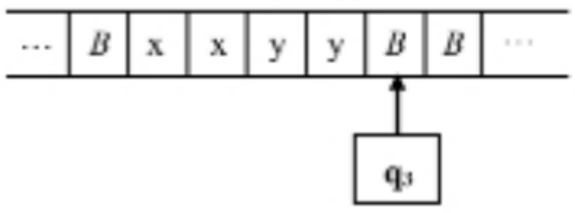


Fig: 2(m)

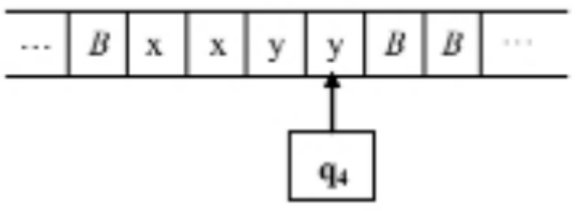


Fig: 2(n)

2nd Part: ***Refer to Question No. 2 (2nd Part) of Long Answer Type Questions.***

5. Define Turing machine. Explain Church's hypothesis. What is Universal Turing machine? **[WBUT 2018]**

Answer:

1st Part: ***Refer to Question No. 3.(a) of Long Answer Type Questions.***

2nd Part:

Church's Hypothesis:

∴ This hypothesis has no practical proof.

Weak form:

A Turing Machine can compute anything that can be computed by digital computer.

Strong form:

A Turing Machine can perform any possible computation.

∴ **Church States that —**

A function on the natural numbers is computable by a human being fallowing an algorithm, ignoring resource limitations, if and only if it is computed by a Turing Machine.

3rd Part:
Universal Turing Machine

∴ Universal Turing Machine is a Turing Machine for all other turing machines.
∴ For example,

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a turing machine and } M \text{ accepts } w\}$$

is Turing Recognizable.
If, M accepts w — Halt & Accept
M rejects w — Halt & Reject
M loops on w — no Halt
For Universal Turing Machine
Input: M = description of some TM
w = an input string for M
Action: — simulate M
— Behave just like M
— Behave just like M (accept, reject or loop)

6. Explain Turing Machine as Decider. **[MODEL QUESTION]**

Answer:
The Turing Machine is eventually comes to a SUCCess halt.
Functions like *NEXT*, *PREV*, *ADD*, etc., i.e., for which such Turing Machine's can be constructed, are called **Primitive Recursive Functions.**
Any function that can be constructed as a finite number of compositions of other primitive recursive functions are themselves primitive recursive functions. For example, we can define the multiplication operation $PROD(m, n)$ as:

$$PROD(m,n) = ADD(m,m),(n-1)\text{times}$$

Integer arithmetic is just one kind of operation that a TM is capable of performing. There are several other operations or "problems" that TM-s are also capable of performing. There is a class of problems like computing arithmetic functions, where the TM "generates" something during its operation. In another class of operations, the TM may "decide" (by coming to a success or a failure halt) about something - the "something" being represented in some way as the input to the machine. An 'YES' answer to the decision question leads to a success halt while a 'NO' answer leads to a failure halt.

A "binary" arithmetic function can be converted to a "trinary" decision problem by conceiving of a TM with three inputs | the numbers to be operated on and the possible result. The TM comes to a success halt if the given results is actually the result of the binary arithmetic function and to a failure halt if it is not.
Under such a circumstance, we say that the **Turing Machine is acting as a Decider**.
For example, suppose L is a Language. The "problem" for a TM could be whether, if given a string on the tape, the TM comes to a SUCCess halt if the string belongs to L and to a FAILure halt if it does not. The concept is not as easy as it sounds.

In our description of the Turing Machine, we have never introduced the surety that a given Turing machine will have to halt (success or failure) in finite time. Indeed, a machine can go on running for an arbitrary amount of time and just looking at the `snapshot' of the machine, it may not be possible to tell whether it will 'ever' come to a halt.

7. Explain: Universal Turing machine. **[MODEL QUESTION]**

Answer:

Consider a Turing Machine M_a such that its input consists of the following:

- The "description" of another machine M_b laid out as a string of 5-tuple with each tuple delimited at the end by a "blank".
- The tape description of the "input given to M_b".

Suppose M_a uses the tape beyond the "input for M_b" as the *working memory*.

We can show that it is possible to design M_a such that it *simulates* M_b, i.e., it works on the input to M_b exactly as M_b would have worked on it.

First, there are two important data stored in a identifiable part of the working memory:

- The current "input of M_b", that is the symbol under the tape head of the "simulated" M_b.
- The current input position of M_b -s head, as an offset from the "input given to M_b".

To start with, the head of M_a is on the first 5-tuple state description of M_b, i.e., the head of M_a is on the tuple whose s_i is the initial state of M_b.

Let s be the s_j (i.e., old-state) value of the current tuple and let p be the current position of the head of simulated M_b (as is written in the "working memory").

The *simulation* by M_a proceeds as per the "program" described below:

1. Store s and p in the working memory
2. If s happens to be a SUCCess halt state of M_b, halt with success; else if s happens to be a FAILure halt state of M_b, halt with failure.
3. Note the current input of M_b (i.e., the symbol at the p-th position of M_b 's tape) and *store it in the finite control* (see Sec-14.5). Let this current input be x.
4. Search for the tuple (s, x, t, ω, d), i.e., whose s_i value is s and the a value is x.
5. Store t value, w and d values of the tuple in finite control
6. Proceed to the p-th position on the tape of M_b and write ω. Make $s \leftarrow t$. Make $p \leftarrow p+1$ if d is "right" else make $p \leftarrow p-1$
7. Goto Step-1

Looking at or description of M_a, we find that no matter what the description of M_b is, *it can be simulated*.

In Formal Language parlance, a TM like M_a is known as a **Universal Turing Machine**.

8. Turing Machine as Enumerator. **[MODEL QUESTION]**

Answer:
When a TM works in that mode, it generates one output after another in a systematic way. The outputs are placed one after the previous one generated, on the infinite tape. In each *step* of operation, the TM adds a new output.
The set of outputs that a given TM generates may be finite or infinite. Even if the output set is infinite, it will have some commonality based on the fact that our given TM has generated it.
Definition: The **Language of a Turing Machine** (when it is an enumerator) is the set of strings that the TM generates as output. For TM M, we often use the notation $L(M)$ to denote the language enumerated by M.

9. State and proof Rice Theorem. **[MODEL QUESTION]**

Answer:
Rice theorem: It states that any non-trivial semantic property of a language which is recognized by a Turing machine is undecidable. A property, P, is the language of all Turing machines that satisfy that property.
If P is a non-trivial property, and the language holding the property, L_p , is recognized by Turing machine M, then $L_p = \{<M> \mid L(M) \in P\}$ is undecidable.

Properties:

- Property of languages, P, is simply a set of languages. If any language belongs to P ($L \in P$), it is said that L satisfies the property P.
- A property is called to be trivial if either it is not satisfied by any recursively enumerable languages, or if it is satisfied by all recursively enumerable languages.
- A non-trivial property is satisfied by some recursively enumerable languages and are not satisfied by others. Formally speaking, in a non-trivial property, where $L \in P$, both the following properties hold:
    - **Property 1** − There exists Turing Machines, M1 and M2 that recognize the same language, i.e. either (<M1>, <M2> ∈ L) or (<M1>,<M2> ∉ L).
    - **Property 2** − There exists Turing Machines M1 and M2, where M1 recognizes the language while M2 does not, i.e. <M1> ∈ L and <M2> ∉ L.

Proof:
Suppose, a property P is non-trivial and $\varphi \in P$.
Since, P is non-trivial, at least one language satisfies P, i.e., $L(M_0) \in P$, ∋ Turing Machine M_0.
Let, w be an input in a particular instant and N is a Turing Machine which follows

- On input x
- Run M on w
- If M does not accept (or doesn't halt), then do not accept x (or do not halt)
- If M accepts w then run M_0 on x. If M_0 accepts x, then accept x.

A function that maps an instance ATM = {<M, w>| M accepts input w} to a N.

- If M accepts w and N accepts the same language as M_0, Then $L(M) = L(M_0) \in p$
- If M does not accept w and N accepts φ, Then $L(N) = \varphi \notin p$

Since A_{TM} is undecidable and it can be reduced to Lp, Lp is also undecidable.

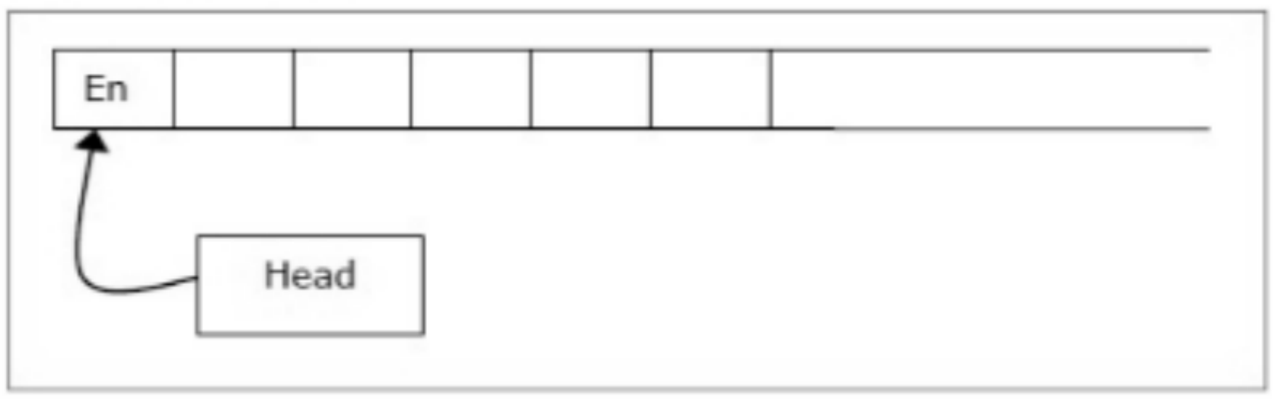
10. Write short notes on: Non-deterministic Turing Machine. [MODEL QUESTION]

Answer:

A non-deterministic Turing machine can be formally defined as a 6-tuple (Q, X, ∑, δ, q_0, B, F) where

- **Q** is a finite set of states
- **X** is the tape alphabet
- ∑ is the input alphabet
- **δ** is a transition function;
  $\delta : Q \times X \rightarrow P(Q \times X \times \{Left\_shift, Right\_shift\})$.
- $\mathbf{q_0}$ is the initial state
- **B** is the blank symbol
- **F** is the set of final states.

A Turing Machine with a semi-infinite tape has a left end but no right end. The left end is limited with an end marker.



It is a two-track tape −

- **Upper track** − It represents the cells to the right of the initial head position.
- **Lower track** − It represents the cells to the left of the initial head position in reverse order.

The infinite length input string is initially written on the tape in contiguous tape cells.
The machine starts from the initial state $\mathbf{q_0}$ and the head scans from the left end marker 'End'. In each step, it reads the symbol on the tape under its head. It writes a new symbol on that tape cell and then it moves the head either into left or right one tape cell. A transition function determines the actions to be taken.
It has two special states called **accept state** and **reject state**. If at any point of time it enters into the accepted state, the input is accepted and if it enters into the reject state, the input is rejected by the TM. In some cases, it continues to run infinitely without being accepted or rejected for some certain input symbols.